On Quantum Nature of Gravity

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In this paper, starting from a new unified formalism of quantum mechanics and thermodynamics developed in Agung Budiyono, ArXiv:quant-ph/0512235 and quant-ph/0601212, we shall derive Einstein general relativity. Gravity will be shown to be not as "fundamental physics", but as "emergent phenomena" of quantum physics, after the latter is decoded in term of geometrical language. In particular, the celebrated Einstein field equation with discrete spectrum of negative definite cosmological constants will be proven to be valid only in the vicinity of stable/marginally stable thermodynamical local equilibrium states. Each cosmological constant characterizes a local thermodynamics equilibrium state. We shall first apply the new approach of quantum-gravity to derive the Bekenstein-Hawking entropy for general space-time, and clarifies its problematic physical meaning. We shall then prove that our reformulation of general relativity does not suffer from the cosmological singularity at the beginning of the universe. The initial universe is shown to be extremely dense, yet finite. Finally, we shall discuss the ontological meaning of space-time quantization, the emergence of Poincare invariance, and showing the realization of Penrose's proposal on gravity induced wave function collapse.

§1. Introduction

The unification of quantum mechanics and general relativity, the so-called quantum-gravity, is perhaps the most ambitious open problem of theoretical physics, today. The former is unbeatable in explaining the experimental data concerning the world of very small and light physical objects, whereas the latter provides the frame work for the physics of space-time, in particular of gravitation, and has been proven experimentally to be very precise in its prediction on the behaviors of the large scale structure of our universe. Yet, although the project has been started soon after the discovery of the quantum mechanics, despite many proposals in the last fifty years: super-string theory, loop quantum gravity etc., to mention two of the most popular,[1] there is no unified theory, so far, that is universally accepted.

One of the great obstacle is indeed inherent in the language used to describe both ingredient theories. In general relativity, physical reality is described by space-time geometry and thus "precise" in nature and "objective-ontological". By contrast, the idea of an objective reality is denied in the orthodox quantum mechanics.[2),3)] The state of a system in the orthodox quantum mechanics is described using matrix density living in a Hilbert space, representing all the possible results of measurements, rather than referring directly to the physical reality itself. Hence, the theory is "epistemological" in nature, and the state of a system is governed by "intrinsic uncertainty".

In particular, the epistemological nature of the language structure of orthodox quantum mechanics assumes a fundamental separation between "the observer" and "the physical system to be observed". Orthodox quantum mechanics then postulates a scenario that the interaction between the two through a measurement will collapse the wave function of the joint-system into one of the pointer states of the observer, thus bringing to us a definite (classical) reality from a vast quantum potentialities, in a random way. This scenario, though is satisfying for all pragmatical purposes,[4] will lead to an obvious and no more avoidable conceptual difficulty if one applies it to our universe as a whole, with no dynamical degree of freedom left for defining an observer. Since any theory of quantum-gravity must say something about cosmology, in fact this constitutes one of the greatest motivation for the study of quantum-gravity, then, any reasonable theory of quantum-gravity should be developed by first curing this "epistemological pathology" of the orthodox quantum mechanics. A new objective-ontological quantum theory which can provide a spontaneous wave function collapse requiring no "external observer" is obviously needed. In fact, the resistance against the epistemological nature of orthodox quantum mechanics is as old as the quantum mechanics itself, as advocated by Einstein, Bell and others.[4)–6)] One of the pillar of this epistemological language structure is the so-called principle of superposition. In contrast to this powerful principle, Einstein field equation which governs the evolution of the space-time geometry is highly non-linear and thus gives no chance to superpositions of solutions.

On the other hand, Einstein general relativity also suffers its own no less serious foundational problem as the epistemological problem of orthodox quantum mechanics above. Namely, Penrose-Hawking singularity theorems say that for a generic space-time and assuming plausible physical conditions, Einstein general relativity predicts singularity points where it must break down, either at the beginning of the universe or at a point inside a black hole.[7)–9)] In particular, the singularity point at the beginning of universe, the so-called cosmological singularity, is unacceptable since it then must give effects to our present and future state of universe, thus "naked" singularity.

All these suggest us that a consistent theory of quantum-gravity might start by making a radical change even in the level of the language used to describe one of the two ingredient physical theories (quantum theory and general relativity), or even to both ingredient theories. Indeed, the history of theoretical physics is fueled by and directed toward finding a more universal language which encompasses wider classes of phenomena as suggested by the success of gauge theories, describing in a unified language all the non-gravitational forces known so far: electromagnetic, weak, and strong forces.[10)] However, when it comes to gravity, one must be careful since the space-time background, on which the language traditionally obtains its causality structure, must now, along with the suggestion of general relativity, be considered as dynamical variable. Applying the standard language of quantization to this diffeomorphism invariant constraint of general relativity will lead to the famous "problem of time", which in particular will prohibit the Nature to have a "unique" and "definite" direction of causality order or arrow of time.[11),12)] On the other hand, some physicist also believe that a consistent quantum theory of gravity might eventually solve the foundational problems of its two ingredient theories spelled above. Namely, one expects that gravity will provide an objective mechanism for wave function collapse,[13] and some sort of quantum fluctuations is expected to rub-off or smeared-out the naked singularity at the beginning of our universe.[14]

Another motivation for the study of quantum-gravity is that, Einstein general relativity in fact can not live alone. To make Einstein field equation works, one must put-in the stress-energy-momentum tensor of some matter-fields, T_{ab}, which unfortunately can not be determined within the theory, except that general relativity requires the tensor to be symmetric and that the energy-momentum of the matter-field to be conserved. Thus, Einstein general relativity is inherently assuming the validity of some other physical theories within the frame work of general relativity.[9] Indeed, Penrose-Hawking singularity theorems as one of the most important prediction of Einstein general relativity, are based on some physical assumptions, the so-called energy conditions, which are not fully developed inside the general relativity. Hence, even since her birth, Einstein general relativity is wandering around, looking for her soul mate to marry with. Since quantum theory claims himself as a theory of matter, then the marriage between general relativity and quantum mechanics is most natural. In fact, historically, it was Einstein who first remarked the necessity to deal with quantum modification of general relativity.[15]

To date, the first "peaceful acquaintance" between quantum mechanics and general relativity is in the thermo-dynamical behaviors of black holes, obtained using the so-called semiclassical quantum-gravity.[16] First, "classical" general relativity predicts that for each of four laws of ordinary thermo-dynamics, there is a similar law that rules the dynamics of black holes. This is done by identifying the area of the black hole's future event horizon, her surface gravity, and black hole's mass, as black hole's entropy, temperature and internal energy, respectively.[17)–19] Moreover, as a result of quantum particle creation effect performed in still "classical" general relativity back ground, a black hole radiates to infinity all species of particles with a perfect black body spectrum.[20] Hence, as long as one does not bother the quantization of geometry of space-time, and exercises the quantum (field) mechanics in static geometrical back ground, a black hole can be considered as an ordinary thermal object. These novel theoretical facts guide us that the sought-after unified theory must reproduce all the thermodynamical behaviors of black holes, and above all, thermo-dynamics might be deeply involved in any formulation of the unified theory of quantum-gravity.[21]

On the other hand, in our previous paper,[22] we have developed a new dynamical theory, which is objective-ontological and yet reproduces all the mathematical rules that govern the orthodox quantum mechanics. This proves, in contrast to the orthodox quantum mechanics, that a quantum system can be described using a language that refers directly to the objective reality that are embedded in every phenomena, the language that is favored by Einstein general relativity. We have also showed that the principle of superposition has no physical nature and can only be considered as a contrived mathematical tool for the sake of calculation. Moreover, in our subsequence paper,[23] even for a single particle system, we showed that the new dynamics can be utilized to prove all the four laws of thermo-dynamics, which tells us that the so-called quantum fluctuations and thermal fluctuations are actually two names for single phenomena. In fact, we showed that the second law of thermodynamics will provide a natural and spontaneous mechanism for the wave function collapse of any closed system, thus necessitating no observer nor environment. For this reason, later on we shall call the new dynamics as quantum-thermo-dynamics. Further, a little effort shows that the Boltzmann-Gibbs-Shannon entropy is indeed linearly proportional to the dynamical surface area of the particle, and extending the dynamics to adopt the principle of special relativity shows that the internal energy of the particle is indeed proportional to the mass of the particle. Then, in the state of stable thermo-dynamics equilibrium, the particle is trapped in a dynamical volume, which thermo-dynamically behaves like a black hole. The surprising point is that, these results are obtained in a flat Minkowskian space-time back ground.

These theoretical facts of the flat space-time quantum-thermo-dynamics, if compared to the black hole thermo-dynamics which are derived in curved space-time, naturally suggests that quantum phenomena which occurs in flat space-time and gravitation phenomena of curved space-time might not be two exclusive phenomena to be unified. But, both might be similar phenomena which is encoded by two different languages: stochastic internal energy potential for the flat space-time quantum-thermo-dynamics and Ricci curvature of a curved space-time for general relativity. Or, after a language decoder has been applied to one of the theories, it might be clear that one is in the subset of the other or at least each contains some part of the other. The programme to quantize gravitation, which is anyway the main goal of the bold subject quantum-gravity, might thus be misguided. A suggestion "to not quantize the gravitation" is also hinted by Jacobson.[24)] Else, in Ref.[25)] Sakharov has suggested that gravity might not be as fundamental as traditionally believed, but is an "emergent phenomena" which is induced by quantum field theory, in the same sense that continuum elasticity theory emerges from molecular physics.[26)] In this way, Shakarov expected to relate Einstein cosmological constant with spectrum of masses of elementary particles.

In this paper, we shall point out the language decoders and apply them to the flat space-time quantum-thermo-dynamics to extract its geometrical content. First, after giving a rather extensive review of the new dynamics developed in Refs.,[22),23)] we shall show that the principle of maximal imaginability which gives the foundation of the new dynamics is equivalent to the principle of equivalence which gives the very foundation of Einstein general relativity. We shall then derive an equation from which limiting ourself to the regime of sufficiently near to stable thermo-dynamics equilibrium states will lead to the celebrated Einstein field equation with discrete spectrum of negative definite Einstein cosmological constant, each corresponding to local stable thermodynamics equilibrium. In fact as expected by Shakarov, the spectrum of Einstein cosmological constants indeed correspond to the spectrum of masses of elementary particles.

In general, the equivalence between the principle of equivalence and the principle of maximal imaginability will guide us that the Lorentzian metric which defines the Einstein tensor is induced quantum mechanically, thus the curved space-time generated by the Lorentzian metric must be considered as "phenomenological" or "effective".[27)] In short, it will be shown that gravity is already included in our reformulation of quantum mechanics, thus it is quantum in nature. We shall then use our new reformulation of general relativity to various problems that are supposed to be relevant for any theory of quantum-gravity. First, we shall derive Bekenstein-Hawking entropy as an almost automatic consequence of the theory, and showing that it is valid for any general space-time structure. Then we shall show that the new formalism of general relativity removes the big bang singularity in the beginning of our universe. Finally we shall discuss the ontology of space-time quantization, the emergence of Poincare invariance, and Penrose's proposal on gravity mediated wave function collapse.

§2. Flat Space-Time Quantum-Thermo-Dynamics

2.1. *Ontological Quantum Theory and Einstein local Causality*

In this section, we shall give a rather extensive review on the quantum-thermo-dynamics in flat space-time, that we developed in Refs.[22),23)] For simplicity we shall consider the dynamics of a single particle with inertial proper mass m. First, let us specify the stage and the players of the dynamics. We assume that the physical stage of the dynamics is a space-time manifold $\mathcal{R}^4$ with a flat Minkowskian metric, $\eta_{ab} = \text{diag}\,(-1,1,1,1)$, defined on it: $\{\mathcal{R}^4, \eta_{ab}\}$. Hence, we assume that η_{ab} is the only physical metric, from which causality structure will be determined and is kept compatible with Einstein local causality postulate that any real velocity must be less than the velocity of light. In this sense, the theory is background dependent. Despite of this, we shall show in the next section that the theory will give Einstein general relativity as its low energy limit. As will be proven later, this is due to a new notion to specify physical states in a nonlocal way, which is explained in the next paragraph.

Fixing a global family of inertial coordinate systems, the state of the particle is then completely specified by two complementary basic realities, or beables:[4)] particle's world event beable, $q = (ct, x, y, z)$, and particle's "imaginability" on the occurrence of events, ρ: $\{q(\lambda), \rho(q'; \lambda)\}$, where ρ is a positive normalized function and λ is some affine parameterization for the evolution of the dynamical system. $\rho(q'; \lambda)$ means that at affine parameter λ, the particle "could be" at space-time point q' with a degree of ρ, though it actually might not be at space-time point q'. It is some sort of "typicality". It is thus of ontological nature referring to a single event, replacing the conventional "epistemological probability" which characterizes a "relative frequency" of occurrence of an event in an ensemble. ρ is thus what Popper and Mermin called as "objective probability".[28),29)] In consequence, any quantity that is derived from ρ will also have an ontological physical meaning. Let us notice that q' inside $\rho(q'; \lambda)$ is not a dynamical variable but only space-time parameter! Since ρ spread all over the whole Minkowskian space-time, then physical state (q, ρ) is non-local. The Minkowskian space-time point q alone, has no more any physical meaning. It is the position of q with respect to ρ which has physical meaning. As will be shown later, ρ will generates a Lorentzian metric g_{ab} thus makes q strictly relational: Namely, any physical event q can only has physical meaning in relation to each other. This will give the source of a diffeomorphism invariance of the theory.

Next, the dynamical evolution of the state of the particle is determined by exercising a modified Hamilton variational principle: "extremizing the accumulation of some Lagrarian, L, while maximizing particle's accumulation of stochastic internal energy, U". The stochastic internal energy is assumed to depend on the particle's imaginability, $U = U[\rho]$ *), and satisfying the following physically motivated postulates:

- (i) Renormalizable or scale free: $U[r\rho] = U[\rho]$, for any real number r. This also means non-local, in the sense that U depends only on the form of ρ not of its intensity.
- (ii) Positive definite along particle's world line/history: $U|_{\mathcal{C}(\lambda)} > 0$, where $\mathcal{C}(\lambda)$ is the particle's world line, and
- (iii) Ehrenfest theorem: the conserved force derived from U, namely $\partial_a U$, must be proportional to the inverse of particle proper mass, m, and, it must also be vanishing averaged over all possible realization, $\int d^4q\,\rho\,\partial_a U = 0$, where $\partial_a = \partial/\partial q^a$ is the ordinary partial differential operator **).

Let us remark first that U is of ontological nature, hence it has no relation with any notion of ensemble. At any instant, it refers to a single event. The last half part of the modified Hamilton variational principle which demands the particle world line to maximize the accumulation of stochastic internal energy at any space-time point parameter q, $H_A(q;\lambda) = \int^{\lambda} d\lambda'\, U(q;\lambda')$, for each degree of freedom, leads to[22]

$$\begin{aligned}\partial_a H_A|_{\mathcal{C}(\lambda)} &= \int^{\lambda} d\lambda' \Big(\frac{\delta U}{\delta\rho}\frac{\partial\rho}{\partial q_a}\Big)\Big|_{\mathcal{C}(\lambda)} = 0,\\ \partial_a^2 H_A|_{\mathcal{C}(\lambda)} &= \int^{\lambda} d\lambda' \Big(\frac{\delta^2 U}{\delta\rho^2}\frac{\partial\rho}{\partial q_a} + \frac{\delta U}{\delta\rho}\frac{\partial^2\rho}{\partial q_a^2}\Big)\Big|_{\mathcal{C}(\lambda)} \leq 0.\end{aligned} \tag{2·1}$$

To understand the above equations, recall that q inside $U(q;\lambda)$ is only a space-time parameter, hence the partial differential operator can smoothly move inside the integral over λ. The above two equations, if combined with the second postulate for U to be definite positive functional of a positive function ρ along the particle's world line, such that $\delta U/\delta\rho|_{\mathcal{C}(\lambda)} > 0$, directly gives

$$\partial_a\rho|_{\mathcal{C}(\lambda)} = 0, \quad \partial_a^2\rho|_{\mathcal{C}(\lambda)} \leq 0. \tag{2·2}$$

Hence, in any degree of freedom, the world line of the particle is always restricted to maximize particle's imaginability. For later reference, let us call Eqs. (2·1) or (2·2) "the principle of maximal imaginability". The above restrictions make the relational meaning of our specification of physical states a bit clearer. Say one has two events q and q', each corresponds to two different maxima of ρ. Then, any local coordinate transformation on q will, due to Eqs. (2·2) generates a global transformation on ρ, thus eventually gives a local transformation on q' as well. This will be shown to keep the relational causality order of events q and q' remains unchanged.

*) The square bracket $[\cdot]$ is used to denote a functional dependence.

**) If not specified otherwise, any index will run from 0 to 3, where index 0 refers to the time part of the corresponding quantity.

Using the postulates (i) to (iii) for U, Eqs. (2·2), and Einstein local causality postulate that particle velocity is always less than the velocity of light, $|v| < c$, the stochastic internal energy, U, can be shown to take the nontrivial form as[22)]

$$U[\rho] = \Upsilon \frac{\spadesuit \mathcal{I}}{m\mathcal{I}}, \quad \spadesuit = -\eta^{ab}\partial_a\partial_b = -\Box, \tag{2·3}$$

where $\mathcal{I} = \rho^{1/2}$, $\Box = \eta^{ab}\partial_a\partial_b$ is the D'Alembertian operator and $\Upsilon = \hbar^2/2$. Rather than repeating the proof of this statement which is given in detail in Ref.,[22)] let us show that U defined in Eq. (2·3) if combined with Einstein local causality postulate indeed satisfies the postulates (i) to (iii). Postulate (i) is of course trivial. Postulate (iii) can be proven easily using partial integration and assuming that ρ is vanishing along the boundary of the dynamical system or at infinity. Hence, the only non-trivial thing is to make U satisfies postulate (ii) which requires U to be positive definite along the particle's world line. To do this, first, in term of $\mathcal{I} = \rho^{1/2}$, the principle of maximal imaginability of Eqs. (2·2) can be written as

$$\partial_a \mathcal{I}|_{\mathcal{C}(\lambda)} = 0, \quad \frac{\partial_a^2 \mathcal{I}}{\mathcal{I}}\Big|_{\mathcal{C}(\lambda)} \leq 0. \tag{2·4}$$

Next, to make the discussion simple let us take an inertial frame such that the particle is moving along the $x-$axis with a velocity v. Then, taking the derivation on $\mathcal{I}$ with respect to t twice, evaluating along the world line, and using the left equation in (2·4), one has

$$\partial_t^2 \mathcal{I}|_{\mathcal{C}(\lambda)} = \partial_t(v\partial_x\mathcal{I})|_{\mathcal{C}(\lambda)} = v^2\partial_x^2\mathcal{I}|_{\mathcal{C}(\lambda)} + \partial_t v \partial_x \mathcal{I}|_{\mathcal{C}(\lambda)} = v^2\partial_x^2\mathcal{I}|_{\mathcal{C}(\lambda)}. \tag{2·5}$$

Using the above relation, U defined in Eq. (2·3) can be put as

$$U|_{\mathcal{C}(\lambda)} = \frac{v^2 - c^2}{c^2}\Upsilon \frac{\partial_x^2 \mathcal{I}}{m\mathcal{I}}\Big|_{\mathcal{I}(\lambda)}. \tag{2·6}$$

Thus, noticing the right inequality in (2·4), U will be positive definite along the particle world line if and only if $|v| < c$, namely if and only if Einstein local causality is valid.

Next, we assume the Lagrarian of the form

$$L = \frac{m}{2}\eta_{ab}v^a v^b, \tag{2·7}$$

which is a direct (special) relativistic generalization of the Lagrarian of a non-relativistic single particle with mass m. We are now ready to evaluate the extremal condition of the modified Hamilton variational principle which says

$$\delta \int d\lambda\, (L - U) = 0, \tag{2·8}$$

and gives us, with the usual fixed end points, the generalized Lagrange equation

$$\frac{d}{d\lambda}\frac{\partial L}{\partial \dot{q}^a} - \frac{\partial L}{\partial q^a} = -\partial_a U. \tag{2·9}$$

Inserting the Lagrarian (2·7), one obtains[22)]

$$\frac{dp^a}{d\lambda}=-\partial^a U,\quad \text{where}\quad p^a=\frac{\partial L}{\partial \dot{q}_a}=mv^a. \tag{2·10}$$

Moreover, from the demand for the conservation of particle's imaginability, which is physically plausible for the closed system we are considering, the above dynamical equation must be coupled to the following continuity equation[22)]

$$\frac{\partial\rho}{\partial\lambda}+\partial_a(\rho\dot{q}^a)=0. \tag{2·11}$$

Eqs. (2·10) and (2·11) with constraints of Eqs. (2·2) thus govern the dynamical evolution of the state of the particle, $(q(\lambda),\rho(q;\lambda))$, in Minkowskian flat space-time background.

Now, let us show that the new dynamics we just developed will give the non-relativistic Schrödinger equation in the limit $v\ll c$.[22)] To to this, let us define a modified action as $S=\int d\lambda\,(L-U)$. Performing again a variation of the particle world line, now with a fixed starting point but with a loose end point, one has[30)]

$$\delta S=\frac{\partial L}{\partial\dot{q}^a}\delta q^a+\int d\lambda\Big(\frac{\partial L}{\partial q^a}-\frac{d}{d\lambda}\frac{\partial L}{\partial\dot{q}^a}-\partial_a U\Big)\delta q^a=p_a\delta q^a, \tag{2·12}$$

where in the second equality we have applied the generalized Lagrange equation of (2·9). One thus has the following important relation

$$\frac{\partial S}{\partial q^a}=p_a. \tag{2·13}$$

On the other hand, using the above relation, one has

$$L-U=\frac{dS}{d\lambda}=\frac{\partial S}{\partial\lambda}+\frac{\partial S}{\partial q^a}\dot{q}^a=\frac{\partial S}{\partial\lambda}+p_ap^a/m. \tag{2·14}$$

Inserting the Lagrarian of (2·7) on the left hand side, one finally obtains the Hamilton-Yacobian formalism of our single particle dynamical problem

$$\frac{\partial S}{\partial\lambda}+p^ap_a/2m+U=0. \tag{2·15}$$

Again, this must be coupled to the continuity equation of (2·11).

Finally, inserting the action-momentum relation of (2·13) into the equations (2·15) and (2·11) one gets a bit more familiar coupled of equations

$$\frac{\partial S}{\partial\lambda}+\frac{1}{2m}\frac{\partial S}{\partial q_a}\frac{\partial S}{\partial q^a}+U=0,\quad \frac{\partial\rho}{\partial\lambda}+\partial_a\Big(\rho\frac{\partial^a S}{m}\Big)=0. \tag{2·16}$$

Now, let us make non-relativistic approximation by taking the formal limit $c\to\infty$. First, the D'Alembertian becomes the usual spatial Laplacian such that U in (2·3) becomes

$$U=-\Upsilon\frac{\partial^2\mathcal{I}}{m\mathcal{I}},\quad \partial^2=\sum_{i=1}^{3}\partial_i^2. \tag{2·17}$$

Further, taking the particle's proper time τ as the affine parameter and noticing that in the limit $c \to \infty$ one has $\tau \to t$, the coupled of equations in (2·16) becomes

$$\frac{\partial S}{\partial t}+\frac{1}{2m}\sum_{i=1}^{3}\frac{\partial S}{\partial q^i}\frac{\partial S}{\partial q^i}+U=0, \quad \frac{\partial \rho}{\partial t}+\sum_{i=1}^{3}\partial_i\Big(\rho\frac{\partial_i S}{m}\Big)=0. \tag{2·18}$$

Notice that the above coupled of equations, combined with eq. (2·17), comprise exactly "the mathematical equations" that define the so-called de Broglie-Bohmian pilot-wave dynamics.[31] Since pilot-wave dynamics is formally equal to the Schrödinger equation (in configuration space), then we have just re-derived non-relativistic Schrödinger equation. However, unlike pilot-wave dynamics which allows all types of trajectories as long as they do not pass through the nodal points of ρ, the world line of our particle is always restricted to maximize ρ by Eqs. (2·2), such that U is kept positive definite. Indeed, this, as shown above makes the dynamics to agree with Einstein local causality postulate, thus eliminating the possibility of superluminal signaling, a very important fact that is lacking in either orthodox quantum mechanics or pilot-wave dynamics. Our formalism thus cures this well-known foundational pathology which plagues the reconciliation between the quantum mechanics and special relativity in a consistent relativistic description of "a single quantum particle", which eventually forcing one to consider "quantum field", instead. Namely, historically it is well-known that quantum field theory which is essentially a multi-particle theory, emerged as an answer that any serious attempt to reconcile the quantum mechanics with special relativity for a single particle always facing the micro-causality problem.[10),32),33)] In this respect, our approach thus raises question on the very basic assumption of quantum field theory. To support this belief, we shall show below that the so-called Klein-Gordon equation is just an approximation to our new dynamics.

Now, let us derive a relation between particle's stochastic internal energy U and proper inertia mass, m, which will be important as we attempt to re-derive Einstein field equation in the next section. Consider the time part of Eq. (2·10)

$$\frac{dp^0}{d\tau}=\frac{\partial U}{c\partial t}, \tag{2·19}$$

where again we have used the particle's proper time as the affine parameterization. Noticing that $p^0 = mv^0 = mc/\sqrt{1-(v/c)^2}$ and expressing t in term of τ one has

$$\frac{dm}{d\tau}=\frac{1}{c^2}\frac{\partial U}{\partial \tau}(1-(v/c)^2). \tag{2·20}$$

Here we have assumed the mass to be dependent on particle's proper time. Hence, in the non-relativistic limit $v \ll c$, one has

$$\frac{dm}{d\tau}\approx\frac{1}{c^2}\frac{\partial U}{\partial \tau}. \tag{2·21}$$

To integrate the above differential equation, one should notice that U contains m from its very definition. Let us give further restriction to Eq. (2·21). Define $\tilde{U} =$

$-(\hbar^2/2)\Box\mathcal{I}/\mathcal{I}$, thus $U = \tilde{U}/m$. Inserting this into Eq. (2·21) one has

$$\frac{dm}{d\tau} \approx \frac{1}{c^2}\Big(\frac{\partial\tilde{U}}{\partial\tau}m - \frac{dm}{d\tau}\tilde{U}\Big)/m^2. \tag{2·22}$$

Assuming that $(d/d\tau)(\ln\tilde{U} - \ln m) \gg 0$, that is the fluctuations of m is neglectable as compared to the fluctuations of $\tilde{U}$, one can neglect the second term on the right hand side. One thus gets

$$m\frac{dm}{d\tau} \approx \frac{1}{c^2}\frac{d\tilde{U}}{d\tau}. \tag{2·23}$$

Integrating and re-arranging one obtains a familiar equation for $\mathcal{I} = \rho^{1/2}$

$$\Box\mathcal{I} + \frac{m^2c^2}{\hbar^2}\mathcal{I} \approx 0, \tag{2·24}$$

which is nothing but the Klein-Gordon equation with a "varying" mass. In the present re-incarnation, un-like its original, the Klein-Gordon equation has a straightforward physical interpretation. "$\mathcal{I}$ is not a physical field to be quantized" as suggested by quantum field theory, but is strictly a quantum wave function of a single relativistic particle. In this case, one has

$$U \approx mc^2/2 \quad \text{or} \quad \tilde{U} \approx m^2c^2/2. \tag{2·25}$$

We shall show in the next section that the above relation will guarantee the validity of Newtonian gravitational physics. Further, assuming a plane wave solution of the type $\mathcal{I} \sim \exp[i(\mathcal{E}t/c - p\cdot q)/\hbar]$ to the Klein-Gordon equation of (2·24), one has the important "energy-momentum" relation

$$\mathcal{E}^2/c^2 - p^2 \approx m^2c^2. \tag{2·26}$$

Yet, the relation is now shown to be just an approximation when $|v| \ll c$ and $(1/m)dm/d\tau$ is ignorable. In general, the above relation is not valid, and must be corrected by non-trivial terms. The correction thus is no more ignorable for high energy elementary particles. Similar claims, developed using loop quantum gravity, are also reported in Refs.[34)–36)] Eq. (2·25) can also be interpreted as giving the origin of mass.

Finally, let us end this subsection to discuss the meaning of classicality in this new dynamics.[22)] This will be relevant as we discuss gravity in the next section. Unlike pilot-wave dynamics which assumes to recover classicality by ignoring the stochastic energy potential, $U = 0$, which is anyway prohibited by postulate (ii), in our new dynamics, classicality is assumed to be recovered by assuming that the quantum force is vanishing

$$\partial^a U|_{\mathcal{C}(\lambda)} = 0. \tag{2·27}$$

Indeed, inserting this into Eq. (2·10), one gets the classical relativistic equation for a free particle

$$\frac{dp^a}{d\lambda} = 0. \tag{2·28}$$

Eq. (2·27) leads to the fact that U is constant of motion

$$\frac{dU}{d\lambda}\Big|_{\mathcal{C}(\lambda)} = v^a\partial_a U = 0. \tag{2·29}$$

Putting the constant as $U_{(i)}$, and recalling the definition of U in (2·3), one then has the following eigenvalue equation

$$\spadesuit\mathcal{I}_{(i)} = -\Box\mathcal{I}_{(i)} = \frac{m_{(i)}}{\Upsilon}U_{(i)}\mathcal{I}_{(i)}, \tag{2·30}$$

where $\mathcal{I}_{(i)} = \rho_{(i)}^{1/2}$ is the eigenfunction belonging to the eigenvalue $U_{(i)}$. It is well-known in the literature of mathematics[37] that the above covariant eigenvalue equation will admit a discrete non-negative eigenvalues $U = U_{(i)}$, $i = 0, 1, 2, \ldots$, which is consistent with the postulate (ii) for U. It is thus clear that $\mathcal{I}_{(i)}$ is nothing but the stationary states of the orthodox quantum mechanics. Thus, in our treatment, classicality and quantum mechanical stationarity are essentially one similar thing. In the next subsection we shall show that quantum mechanical stationary state is also equivalent to thermodynamical equilibrium state. Hence in these states, the stochastic internal energy and thus the total energy must be constant of motion and can take only discrete spectrum of values. Noticing this fact, Eq. (2·25) then becomes

$$m_{(i)} = \frac{2}{c^2}U_{(i)}, \tag{2·31}$$

giving us a discrete spectrum of masses of elementary particles. Let us notice that inserting the expression for $U_{(i)}$ in Eq. (2·31) into Eq. (2·30) one will recover again the Klein-Gordon equation of (2·24). In other words, the Klein-Gordon equation with a constant fixed mass describes a stationary or classical state.

Moreover, since $\mathcal{I}_{(i)}$ makes a complete set of orthonormal functions,[37] one can use it to expand the initial imaginability $\mathcal{I} = \rho^{1/2}$ as

$$\mathcal{I} = \sum_i r_{(i)}\mathcal{I}_{(i)}. \tag{2·32}$$

where $r_{(i)}$ is some complex function. This is just the superposition principle of the orthodox quantum mechanics. Yet, in our formalism it has obviously only a formal mathematical meaning, without physical nature.[22]

Another interesting fact of our formalism is that, unlike the orthodox quantum mechanics or pilot-wave dynamics which predicts that a particle resting inside a box can be observed anywhere with a finite probability, our new dynamics predicts that the allowable places or "position" for a rest particle inside a box must take discrete spectrum.[22] This is due to the principle of maximal imaginability of Eqs. (2·2) that the position of the particle must be the one that maximizes the stationary ρ in the box.

2.2. *Thermodynamics and Spontaneous Wave Function Collapse*

Let us now discuss some relevant thermodynamical behaviors of our single particle dynamics which will later be useful for the re-derivation of Einstein general relativity. Again, a complete treatment is given in Ref.[23] First, let us define a dynamical stable state $(q(t),\rho(q';t))$ as follows

$$\partial_a U|_{\mathcal{C}(\lambda)}=0,\quad \partial_a^2 U|_{(C)(\lambda)}\geq 0, \tag{2·33}$$

whose physical meaning is obvious. Namely, we restrict the world line of our single particle to always take the stable minimum or marginal stable point of stochastic internal energy potential, U, at any degree of freedom. Later on we shall spoilt ourself to simply refer to it as stable. We shall show that this condition will lead to the Maxwell-Boltzmann-Gibbs canonical distribution for ρ. Since the reasoning that leads to this conclusion will be of importance when we re-derive Einstein field equation, we shall repeat it here. First, from the renormalizability postulate for U, one has $\partial_a U[r\rho]=\partial_a U[\rho]$, for any real constant r. One can therefore write the quantum force $\partial_a U$ as[22),23)]

$$\partial_a U[\rho]=\frac{1}{\rho^s}(a_0+a_1\partial_a+a_2\partial_a^2+a_3\partial_a^3+\dots)\rho^s, \tag{2·34}$$

where s and a_i, $i=1,2,3,\dots$, are arbitrary real numbers to be determined later. Using the following fact,

$$\frac{\partial_a^n\rho^s}{\rho^s}\Big|_{\mathcal{C}(\lambda)}=s\frac{\partial_a^n\rho}{\rho}\Big|_{\mathcal{C}(\lambda)}, \tag{2·35}$$

which can be shown easily using induction and utilizing the left equation in (2·2) to be valid for any positive integer n, one has

$$\partial_a U[\rho]|_{\mathcal{C}(\lambda)}=a_0+\frac{s}{\rho}(a_1\partial_a+a_2\partial_a^2+a_3\partial_a^3+\dots)\rho\Big|_{\mathcal{C}(\lambda)}. \tag{2·36}$$

The left equation in (2·33) imposes the right hand side of Eq. (2·36) to be vanishing. Keeping in mind Eqs. (2·2) and the fact that $\partial_a^n\rho|_{\mathcal{C}(\lambda)}$, for $n\geq 3$, are fluctuating between positive and negative value for general actual world line $\mathcal{C}(\lambda)$, $\partial_a U|_{\mathcal{C}(\lambda)}=0$ can then be accomplished by imposing $a_0=0$, $a_j=0$ for $j\geq 2$ and a_1 is arbitrary, yet non-vanishing. One therefore has

$$\partial_a U[\rho]=a_1\frac{\partial_a\rho^s}{\rho^s}. \tag{2·37}$$

One can then prove easily that $s=1$ is the only unique value that satisfies the postulate (iii) for U, that is the Ehrenfest theorem[23)]

$$\partial_a U[\rho]=a_1\frac{\partial_a\rho}{\rho}. \tag{2·38}$$

Now, taking partial derivation to both sides and again using the left equation in (2·2) one gets

$$\partial_a^2 U|_{\mathcal{C}(\lambda)}=a_1\frac{\partial_a^2\rho}{\rho}\Big|_{\mathcal{C}(\lambda)}+a_1\Big(\frac{\partial_a\rho}{\rho}\Big)^2\Big|_{\mathcal{C}(\lambda)}=a_1\frac{\partial_a^2\rho}{\rho}\Big|_{\mathcal{C}(\lambda)}. \tag{2·39}$$

Comparing the above equation to the right inequality in (2·33), keeping in mind the right inequality in (2·2) one concludes that a_1 must be non-positive, $a_1 \leq 0$. For dynamically stable states, one can thus finally writes[23)]

$$-\beta\,\partial_a U = \frac{\partial_a \rho}{\rho}, \quad \beta \geq 0. \tag{2·40}$$

Next, integrating Eq. (2·40) one directly obtains

$$\rho = \frac{1}{Z}\exp(-\beta U), \tag{2·41}$$

where Z is a normalization constant, or the so-called partition function. The expression of (2·41) must remind us to Maxwell-Boltzmann-Gibbs (MBG) canonical distribution by identifying β as the inverse of temperature $\beta = 1/(k_B T)$, where k_B is the Boltzmann constant and T is temperature. ρ given in Eq. (2·41) must then correspond to the "thermodynamically stable equilibrium state". This is our starting point to formulate the equilibrium thermodynamics for our single particle system. In fact, in Ref.[23)] we have shown that the zeroth, first, and the third law of equilibrium thermodynamics can be proven in simple manner. In particular, as will be relevant later in the discussion of gravity, the first law of thermodynamics can be written as

$$\delta\bar{U} = T dS_e + \text{"work"}, \tag{2·42}$$

where $\bar{U} = \int d^4q\, U\rho$, and S_e is nothing but the Boltzmann-Gibbs-Shannon (BGS) entropy given by

$$S_e[\rho] = -k_B \int d^4q\, \rho \ln \rho. \tag{2·43}$$

Thus $\bar{U}$ must be considered as thermodynamical internal energy. This is the reason why we named U as the stochastic internal energy from the beginning. The term "work" in Eq. (2·42) counts the work done by the system as the response of parameter variation.

Let us discuss the second law of thermodynamics as it will play important role later on. Differentiating both sides of Eq. (2·43) with respect to affine parameter and imposing the continuity equation of Eq. (2·11), one has

$$\frac{dS_e}{d\lambda} = k_B \int d^4q\, \partial_a(\rho \dot{q}^a)(\ln\rho + 1). \tag{2·44}$$

Integrating by parts twice, assuming that ρ is vanishing at the boundary or infinity, one finally obtains

$$\frac{dS_e}{d\lambda} = k_B \int d^4q\, \rho\, \theta = k_B \langle\theta\rangle_\rho, \tag{2·45}$$

where $\theta(q;\lambda) = \partial_a v^a(q;\lambda)$ is the four velocity divergence, and $\langle\cdot\rangle_\rho$ denotes an averaging process over ρ.

One thus needs to investigate the nature of the velocity divergence field $\theta(q;\lambda)$. To do this, fixing a space-time point q and integrating Eq. (2·10) over the affine parameter, one obtains four velocity field

$$v^a(q;\lambda) = -\frac{1}{m}\int^{\lambda} d\lambda'\,\partial^a U(q;\lambda'). \tag{2·46}$$

Further taking divergence to both sides one has

$$\theta = -\frac{1}{m}\partial_a \int^{\lambda} d\lambda' \partial^a U = -\frac{1}{m}\Box\int^{\lambda} d\lambda'\,U = -\frac{1}{m}\Box H_A|_{\mathcal{C}(\lambda)}, \tag{2·47}$$

where, we have assumed that m is constant of motion.

Now, using Eq. (2·47), let us prove that the four velocity divergence θ is always non-negative along the particle's world line. To do this, without losing generality, one can again pick up a specific frame in which the particle is moving along, say, $x-$axis with a velocity v. Proceeding in the same way as in the previous subsection, taking the time derivative to H_A twice, and imposing the upper equation in (2·1), one has

$$\frac{\partial^2 H_A}{\partial t^2}\Big|_{\mathcal{C}(\lambda)} = \frac{\partial^2 H_A}{\partial x^2}\Big|_{\mathcal{C}(\lambda)} v^2. \tag{2·48}$$

Using this fact, Eq. (2·47) can be put as

$$\theta = -\frac{1}{m}\frac{\partial^2 H_A}{\partial x^2}\Big(\frac{c^2-v^2}{c^2}\Big)\Big|_{\mathcal{C}(\lambda)}. \tag{2·49}$$

Hence, imposing the lower inequality in (2·1) and assuming that Einstein local causality postulate is valid, $|v|<c$, the right hand side is always never negative, $\theta|_{\mathcal{C}(\lambda)} \geq 0$. Inserting this fact into Eq. (2·45) one finally obtains

$$dS_e/d\lambda \geq 0. \tag{2·50}$$

On the other hand, it is well-known that the MBG canonical distribution of stable thermodynamics equilibrium state maximizes the BGS entropy,[38] thus giving an upper bound. Hence, this fact just completes our proof of the second law that entropy of any closed system is never decreasing, that is monotonically increasing for non-equilibrium states and constant of motion for stable equilibrium state. Since, $\theta = 0$ corresponds to $dS_e/d\lambda = k_B\langle\theta\rangle = 0$, then $\theta = 0$ must characterize a stable thermodynamics equilibrium state. In the next section, we shall show the process that leads to the vanishing of θ in term of geometry. Moreover, since the non-negativity of θ along the particle's world line is guaranteed and guarantees that $|v|<c$, then the second law and Einstein's local causality postulate are equivalent to each other. This is an intuitive result since both are basically telling the unique direction of time[*)].

We have just then proven the second law from the first principle of the dynamics. One can thus conclude that the second law will drag the system toward thermodynamical equilibrium in a spontaneous manner. Since in stable thermodynamical equilibrium we have $\partial_a U|_{\mathcal{C}(\lambda)} = 0$, the dynamics is classical in nature or quantum

[*)] Traveling faster than light is basically moving backward to the past which is causally absurd.

mechanically stationary. Hence, the second law thus provides a spontaneous mechanism to collapse any initial wave packet into one of the system's stationary or classical states, without any requirement of observer or environment!

Finally, let us discuss one more important property of our dynamics concerning the relation between the entropy and dynamical volume. For physical simplicity, let us consider the non-relativistic regime. In this case, space and time decouples such that one has $\rho(q) = \rho_s(x, y, z)\rho_t(t)$, and the time part has reached a thermodynamics equilibrium state. Thus, one can write $S_e = -k_B \int d^3q\ \rho_s \ln \rho_s - ck_B \int dt\ \rho_t \ln \rho_t$, where the second term on the right hand side is constant. Differentiating with time, and proceeding as before, writing ρ_s again as ρ, one thus has[23)]

$$\frac{dS_e}{dt} = k_B \int dq^3\ \theta\rho, \tag{2·51}$$

where θ is now the usual (spatial) three velocity divergence $\theta = \sum_i \partial v^i/\partial q^i$.

Now let us define dynamical spatial volume as

$$\Delta V = \prod_{i=1}^{3} \Delta q^i. \tag{2·52}$$

Differentiating with time t and after some simple manipulation one obtains[23)]

$$\frac{d(\Delta V)}{dt} = \Delta V\theta. \tag{2·53}$$

On the other hand, the non-relativistic version of the entropy of (2·51) can be discretized in the form

$$\frac{dS_e}{dt} = k_B \lim_{\sup\{\Delta V_{(j)}\}\to 0} \sum_j \Delta V_{(j)}\rho(q^i_{(j)})\theta(q^i_{(j)}), \tag{2·54}$$

where the index inside a bracket is a partition index. Inserting Eq. (2·53) into the above equation one obtains an important relation

$$\frac{dS_e}{dt} = k_B \lim_{\sup\{\Delta V_{(j)}\}\to 0} \sum_j \frac{d(\Delta V_{(j)})}{dt}\rho(q^i_{(j)}). \tag{2·55}$$

The above relation physically says that the rate of increasing of the BGS entropy is equal to the average rate of the increasing of infinitesimal dynamical volume. The second law thus guarantees that the average of the infinitesimal dynamical volume is never decreasing. We shall prove in the next section that this simple fact will also give us a linear relation between entropy and dynamical surface area, which as discussed in the previous section, is supposed to be the characteristic of any quantum theory of gravity. We shall thus prove the so-called "area law" that the dynamical surface area is never decreasing,[23)] shown to be valid for black hole event horizon by Hawking.[17)]

To conclude, we thus just have proven that quantum theory and thermodynamics can be described in a unified language which is objective-ontological in nature.

The traditionally called quantum and thermal fluctuations are thus one single object described in two different languages. This is the reason why we call the new dynamics as the flat space-time quantum-thermo-dynamics, or simply quantum-thermo-dynamics. In particular, we have shown that the so-called superposition principle in orthodox quantum mechanics is formal mathematical with no physical reality. Moreover the new dynamics shows that the second law of thermodynamics provides a natural and spontaneous mechanism for the occurrence of wave function collapse without any external observer or environment. This suggests that the language of the quantum-thermo-dynamics is in favor of Einstein realism, thus appropriate for quantizing gravity or developing quantum theory for cosmology. Rather than proceeding in this way, we shall show in the next section that gravity has already been included in the above flat space-time quantum-thermo-dynamics. Einstein general relativity will be shown as the remnant of quantum-ness in the vicinity of stable thermodynamics equilibrium states.

§3. On Dynamical Phenomenology of Curved Space-Time

Despite our formulation of the quantum-thermo-dynamics above assumed a preferred coordinate system, that is a global family of inertial frames, all the important physical results, including its thermo-dynamical behaviors, are free of any coordinate representation.[22),23)] This hints us that all those physical results are referring to some coordinate free geometrical objects. We shall prove in this section that our feeling is indeed correct. We shall show that the principle of maximal imaginability of Eqs. (2·1) or (2·2) which governs the quantum-thermo-dynamics, is equivalent to the principle of equivalence which gives the very foundation of the general relativity.

3.1. *Dynamical Origin of Principle of Equivalence and Emergent Lorentzian Space-time*

Let us take a co-moving coordinate frame along the particle's world line, $q(\lambda) = (c\tau(\lambda), 0, 0, 0)$, where τ is particle's proper time. In this frame, the dynamical equation of (2·10) then becomes

$$c^2\frac{d^2\tau}{d\lambda^2} = \frac{1}{m}\frac{\partial U}{\partial\tau}. \tag{3·1}$$

Multiplying both sides with $d\tau$, one has

$$c^2\frac{d\tau}{d\lambda}\frac{d^2\tau}{d\lambda^2}d\lambda = \frac{c^2}{2}\frac{d}{d\lambda}\Big(\frac{d\tau}{d\lambda}\Big)^2 d\lambda = \frac{1}{m}dU. \tag{3·2}$$

Integrating the above equation, one then obtains

$$c^2\Big(\frac{d\tau}{d\lambda}\Big)^2 = \frac{2}{m}U|_{\mathcal{C}(\lambda)} + \text{ some constant.} \tag{3·3}$$

On the other hand, it is always possible to find a new coordinate system $q' = q'(q)$, in which, the left hand side of the above equation can be written as

$$c^2\Big(\frac{d\tau}{d\lambda}\Big)^2 = -g_{ab}v'^a v'^b, \quad v'^a = dq'^a/d\lambda, \tag{3·4}$$

where g_{ab} is some Lorentzian metric to be determined. g_{ab} is then related to the flat Minkowskian metric as $-c^2(d\tau/d\lambda)^2 = \eta_{ab}v^a v^b = g_{ab}v'^a v'^b$, or

$$g_{ab} = \eta_{cd}\frac{\partial q^c}{\partial q'^a}\frac{\partial q^d}{\partial q'^b}. \tag{3·5}$$

Eq. (3·3) and (3·4) then simply tell us that if the world line of the particle maximizes its accumulation of stochastic internal energy $H_A = \int d\lambda\, U$, that is the principle of maximal imaginability of Eqs. (2·1) or (2·2) is valid, then the particle's world line will also maximize $-\int d\lambda\, g_{ab}v'^a v'^b$. It is well-known that maximizing this last quantity will lead to a geodesic equation on a curved space-time manifold characterized by the Lorentzian metric g_{ab}[39]

$$v'^b\nabla'_b v'^a = \frac{dv'^a}{d\lambda} + \Gamma^a_{\ bc}v'^b v'^c = 0, \quad \Gamma^a_{\ bc} = \frac{1}{2}g^{ad}(\partial'_b g_{cd} + \partial'_c g_{bd} - \partial'_d g_{cb}). \tag{3·6}$$

where, in coordinate basis representation one has $d/d\lambda = v'^b\partial'_b$, and ∇'_a, $\Gamma^a_{\ bc}$ are the covariant derivative and the Levi-Civita connection, respectively, associated with the Lorentzian metric g_{ab}. Conversely, it is easy to show that inserting Eqs. (3·3) and (3·4) into Eq. (3·6) will lead us to arrive back at Eq. (2·10). Notice that using the coordinate transformation generated by Eq. (3·5), the Levi-Civita connection can be put as

$$\Gamma^a_{\ bc} = \frac{\partial q'^a}{\partial q^d}\frac{\partial^2 q^d}{\partial q'^b\partial q'^c}. \tag{3·7}$$

Hence, we have just proven by suitable coordinate transformation that the quantum-thermo-dynamical equation in flat space-time of (2·10) is equivalent to the geodesic equation of (3·6) in a curved space-time characterized by g_{ab}, given by Eq. (3·4). The correspondence is facilitated by the principle of maximal imaginability and Eqs. (3·3), (3·4). In other words, the principle of maximal imaginability enables us to choose a new coordinate system such that the particle is seen as a free falling body. Surprisingly, this is just the content of "equivalence principle" which gives the very foundation of Einstein general relativity.[40] The purely quantum mechanical object of stochastic internal energy U thus generates the Lorentzian metric g_{ab} of a curved space-time manifold. For later reference, let us call the curved space-time manifold characterized by the Lorentzian metric g_{ab} as the "effective" curved space-time manifold. The effective curved space-time manifold can thus be regarded as "phenomenological object", or "emergent object",[27] which is obtained by adjusting the particle's proper time with its stochastic internal energy as in Eq. (3·3). Thus, physical time is, in this sense, internally defined. Notice that since Eq. (3·6) is purely geometric and thus coordinate free, then the original dynamical equation of Eq. (2·10) is essentially also a coordinate free geometrical equation. Here on, any symbols with dashed sign, ('), is used to refer that it is living in the new coordinate system of effective curved space-time with Lorentzian signature, g_{ab}, whereas a symbol without dashed sign is referring to the old flat Minkowskian space-time representation of the dynamics.

Hence, at any instant of λ, there is a correspondence between the state of the particle in flat space-time characterized by the internal stochastic energy U and the particle's world line in the old coordinate system v^a, and the state of the particle as a geodesic v'^a in an effective curved space-time, spanned by a new coordinate and characterized by a Lorentzian metric g_{ab} as

$$(U[\rho], v^a) \sim (g_{ab}, v'^a). \tag{3·8}$$

The above correspondence is one version of our language decoder which will be up-dated in the next subsection. Notice that the correspondence is local. This correspondence then tells us that in general, the Lorentzian metric g_{ab} of the effective curved space-time is varying along with the variation of the internal stochastic energy U during the dynamical evolution. Recall that U is not a type of pre-assigned potential, but varies in the evolution of the dynamics in accordance with the continuity equation of (2·11). In consequence, considering the particle's world history $C(\lambda)$ as a geodesic, its effective curved space-time manifold along which the geodesic moves, is no longer fixed, but is also a "dynamical variable". In the specific case of the stable thermo-dynamics equilibrium states, however, since the stochastic internal energy U and the particle's velocity are locally constants of motion as discussed in the previous section, then is also the Lorentzian metric g_{ab}. Moreover, the quantization of the values of U into a discrete allowable spectrum $\{U_{(i)}\}$ must lead also to some sorts of quantization of the Lorentzian metric $\{g^{(i)}_{ab}\}$. The above physical picture will be clearer as we proceed in the next subsections.

Let us investigate the physical nature of the constant term that appears on the right hand side of Eq. (3·3). Inserting Eq. (3·4), one has

$$-g_{ab}v'^a v'^b = \frac{2}{m}U|_{\mathcal{C}(\lambda)} + D|_{\mathcal{C}(\lambda)}, \tag{3·9}$$

where $D|_{\mathcal{C}(\lambda)}$ denotes the yet undetermined constant. Now, to have a complete correspondence expressed in Eq. (3·8), one must specify the unique stochastic internal energy, U, which locally corresponds to the flat space-time characterized by Minkowskian metric, $g_{ab} = \eta_{ab}$. Recall that in locally flat space-time the Levi-Civita connection is vanishing,

$$\Gamma^a_{bc} = 0. \tag{3·10}$$

Inserting this into the geodesic equation of (3·6), one has

$$\frac{dv'^a}{d\lambda} = 0. \tag{3·11}$$

Let us decodes it back into the language of the flat space-time quantum-thermodynamics. Using the inverse coordinate transformation $q = q(q')$, one has

$$\frac{dv^a}{d\lambda} = \frac{d}{d\lambda}\Big(\frac{\partial q^a}{\partial q'^b}\frac{dq'^b}{d\lambda}\Big) = \frac{\partial q^a}{\partial q'^b}\frac{d^2q'^b}{d\lambda^2} + \frac{\partial^2 q^a}{\partial q'^b \partial q'^c}\frac{dq'^b}{d\lambda}\frac{dq'^c}{d\lambda}. \tag{3·12}$$

Eq. (3·11) will make the first term on the right side vanishing. Moreover, using the fact that the Levi-Civita connection is locally vanishing, Eq. (3·7) leads to

$\partial^2 q^a/\partial q'^b \partial q'^c = 0$. This will also make the second term on the right hand side vanishing. Eq. (3·11) thus in flat space-time corresponds to

$$dv^a/d\lambda = 0. \tag{3·13}$$

Inserting this into the dynamical equation in the old flat space-time coordinate system of Eq. (2·10), one finally has

$$\partial^a U|_{\mathcal{C}(\lambda)} = 0. \tag{3·14}$$

The above is nothing but a condition for stationary or classical states. As discussed in previous section, the stochastic internal energy will be constant of motion and takes a discrete spectrum, $U = U_{(i)},\ i = 1, 2, 3 \ldots$, which are the eigenvalues of the operator $\spadesuit = -\Box$. Therefore, the locally flat space-time must be recovered when the stochastic internal energy along the particle's world history, $U|_{\mathcal{C}(\lambda)}$, is equal to one of these positive definite constant, $U_{(i)}$. Hence, in locally flat space-time one must impose

$$-\eta_{ab} v'^a v'^b = \frac{2}{m} U_{(i)} + D|_{\mathcal{C}(\lambda)}. \tag{3·15}$$

Inserting Eq. (3·15) back into Eq. (3·9), one finally obtains

$$-\frac{m}{2} g_{ab} v'^a v'^b + U_{(i)} = U|_{\mathcal{C}(\lambda)} - \frac{m}{2} \eta_{ab} v'^a v'^b. \tag{3·16}$$

The above equation can be interpreted that something is being conserved locally, in the transformation of coordinate which takes the flat space-time quantum-thermo-dynamics onto the free falling particle in an effective curved space-time with a Lorentzian metric g_{ab}. Let us remark that Eq. (3·6) together with Eq. (3·16) have already given us a complete information on the geometrical content of the quantum-thermo-dynamics in term of space-time Lorentzian metric, g_{ab}. Nevertheless, in the next subsection we shall proceed to develop a physically more intuitive correspondence which relates the stochastic internal energy, U, of the flat space-time quantum-thermo-dynamics, with the Ricci curvature, R_{ab}, of the corresponding effective curved space-time. This then allows us to directly discuss the ontology offered by Einstein general relativity.

Finally, let us put the above conservation law into a form which is more intuitive and of great importance later on. To do this, let us define a second rank tensor, U_{ab}, as

$$U|_{\mathcal{C}(\lambda)} = U_{ab} v'^a v'^b, \tag{3·17}$$

where v'^a is the tangent of the time-like geodesic, $\mathcal{C}(\lambda)$, of the effective curved space-time manifold. It is obvious that U_{ab} should be interpreted as the "stochastic internal" stress-energy-momentum tensor of the particle. Moreover, it is also obvious that U_{ab} and any tensors below defined in this way, unlike U, are local quantities in space-time, to be always evaluated along the particle's geodesic. Inserting this into Eq. (3·9), one has

$$-g_{ab} v'^a v'^b = \frac{2}{m} U_{ab} v'^a v'^b + D|_{\mathcal{C}(\lambda)} \tag{3·18}$$

Taking the derivative to both sides, using the geodesic equation of (3·6) and noticing the fact that $D|_{\mathcal{C}(\lambda)}$ is a constant of motion, one obtains

$$v'^c\nabla'_c(U_{ab}v'^a v'^b) = v'^c v'^a v'^b \nabla'_c U_{ab} = -\frac{m}{2}v'^c v'^a v'^b \nabla'_c g_{ab} = 0, \tag{3·19}$$

where the last equality is due to the compatibility between the covariant derivative and the Lorentzian metric, $\nabla'_c g_{ab} = 0$, which is generally valid. Hence, since Eq. (3·19) must be valid for any general geodesic, one has $\nabla'_c U_{ab} = 0$. Contracting the first two indices, one finally obtains

$$\nabla'^a U_{ab} = 0, \tag{3·20}$$

which can be interpreted as a statement of local conservation of "stochastic internal" energy-momentum. This last fact will play an important ingredient while we re-derive the Einstein field equation in the next subsection.

3.2. *On Einstein Field Equation: Quantum Origin of Gravity*

Let us proceed to discuss the physical status of the Einstein field equation. First, taking the derivative to both sides of Eq. (2·47), one has

$$\frac{d\theta}{d\lambda} = -\frac{1}{m}\Box U|_{\mathcal{C}(\lambda)}. \tag{3·21}$$

On the other hand, using the new coordinate system which takes us to the effective curved space-time manifold, the velocity divergence can be written as

$$\theta = \partial_a v^a = \frac{\partial q'^b}{\partial q^a}\frac{\partial}{\partial q'^b}\Big(v'^c\frac{\partial q^a}{\partial q'^c}\Big) = \frac{\partial q'^b}{\partial q^a}\frac{\partial q^a}{\partial q'^c}\frac{\partial v'^c}{\partial q'^b} + v'^c\frac{\partial q'^b}{\partial q^a}\frac{\partial^2 q^a}{\partial q'^b\partial q'^c}. \tag{3·22}$$

Using the fact that $\partial q'^b/\partial q'^c = \delta^b_{\ c}$, and noticing the definition of the Levi-Civita connection of Eq. (3·7), one finally obtains

$$\theta = \partial_a v^a = \partial'_c v'^c + \mathit{\Gamma}^b_{\ bc} v'^c = \nabla'_c v'^c, \tag{3·23}$$

namely, velocity divergence is coordinate free.

One can now works in curved coordinate system q' and employing the so-called Raychaudhuri's equation which says that, for any smooth congruence of time-like geodesic in curved space-time with Lorentzian metric g_{ab}, thus satisfies Eq. (3·6), the derivative of the velocity divergence can be put as follows[39)]

$$\frac{d\theta}{d\lambda} = -\frac{1}{3}\theta^2 - \sigma^2 - R_{ab}v'^a v'^b, \tag{3·24}$$

where $\sigma^2 = \sigma^{ab}\sigma_{ab}$ is the square of the shear and R_{ab} is the Ricci curvature tensor of the effective curved space-time. Comparing Eqs. (3·21) and (3·24), one obtains the following general relation

$$\frac{1}{m}\Box U = \frac{1}{3}\theta^2 + \sigma^2 + R_{ab}v'^a v'^b. \tag{3·25}$$

The above equation thus gives the geometrical content of the quantum-thermo-dynamics of our single particle, in term of Ricci curvature of the corresponding effective curved space-time. Namely, at any instant, given the solution of the flat space-time quantum-thermo-dynamics, $(q(\lambda), \rho(q';\lambda))$, one can deduce the Ricci curvature R_{ab} of the corresponding effective curved space-time, in which the particle can be seen to move geodesic-ally. In other words, at any instant of dynamical evolution, one has a one to one correspondence between particle's state in flat space-time characterized by the stochastic internal energy potential and the particle's world line in flat space-time, $(U[\rho], v^a)$, and the particle's state in the corresponding effective curved space-time characterized by the Ricci curvature tensor and particle's geodesic, (R_{ab}, v'^a), as

$$(U[\rho], v^a) \sim (R_{ab}, v'^a). \tag{3·26}$$

This is the up-graded version of our language decoder. Needless to say, the content of Eq. (3·25) is similar to Eq. (3·16).

Let us discuss the case when the particle is sufficiently near to one of its stable thermo-dynamics equilibrium states. In this case, one can thus regard θ and σ to be neglectable small corrections such that Eq. (3·25) reduces into

$$\frac{1}{m}\Box U|_{\mathcal{C}(\lambda)} = R_{ab}v'^a v'^b. \tag{3·27}$$

First, recall that as discussed in the previous section, the states of "stable" thermo-dynamics equilibrium are given by the condition[23)]

$$\partial_a U|_{\mathcal{C}(\lambda)} = 0,\ \ \partial_a^2 U|_{\mathcal{C}(\lambda)} \geq 0.$$

This leads to the following important relation

$$\partial_a U = -k_B T\frac{\partial_a \rho}{\rho}.$$

Taking the divergence to both sides, and evaluating along the particle's world line, one obtains[23)]

$$\begin{aligned}\Box U|_{\mathcal{C}(\lambda)} = \eta^{ab}\partial_a\partial_b U|_{\mathcal{C}(\lambda)} = -k_B T\frac{\Box\rho}{\rho}\Big|_{\mathcal{C}(\lambda)} &= -\frac{2mk_BT}{\Upsilon}\Upsilon\frac{\Box\mathcal{I}}{m\mathcal{I}}\Big|_{\mathcal{C}(\lambda)}\\ = \frac{2mk_BT}{\Upsilon}U\Big|_{\mathcal{C}(\lambda)} &= \frac{2}{\pi\Lambda_B^2}U|_{\mathcal{C}(\lambda)},\end{aligned} \tag{3·28}$$

where, in deriving the second and third equalities we have used the left equation in (2·2), and $\Lambda_B = \sqrt{\Upsilon/\pi mk_BT}$ is the so-called "thermal" de-Broglie wavelength. Notice that in this case, since $\partial_a U|_{\mathcal{C}(\lambda)} = 0$ thus $U|_{\mathcal{C}(\lambda)} = U_{(i)}$, where $U_{(i)}$ is one of the positive definite eigenvalues of $\spadesuit = -\Box$. Hence, in stable thermodynamics equilibrium states U becomes a valley whose bottom is given by $U_{(i)}$. This situation is pictorially clear if one takes non-relativistic regime, $|v| \ll c$, such that the above equation becomes $\partial^2 U|_{\mathcal{C}(\lambda)} = (2/\pi\Lambda_B^2)U|_{\mathcal{C}(\lambda)}$.

It is thus evident that in the state "sufficiently near" to stable thermo-dynamics equilibrium, the gradient of the stochastic internal energy must "not" be vanishing, yet it must still be sufficiently small, and moreover, one must keep the right inequality in Eq. (2·33) to hold

$$\partial_a U|_{\mathcal{C}(\lambda)} \approx 0, \;\; \partial_a^2 U|_{\mathcal{C}(\lambda)} \geq 0. \tag{3·29}$$

Proceeding in the same manner as for the case of stable thermo-dynamics equilibrium states discussed in the previous section, the conditions of (3·29), if combined with the renormalizability property of U, the principle of maximal imaginability of Eqs. (2·2), and the Ehrenfest theorem, will lead to

$$\partial_a U = e_a - \Omega \frac{\partial_a \rho}{\rho}. \tag{3·30}$$

The non-vanishing of the first term on the right hand side is guaranteed by the left equation in (3·29). This is obvious, since evaluating along the particle world line will give $\partial_a U|_{\mathcal{C}} = e_a|_{\mathcal{C}}$, due to the principle of maximal imaginability of (2·2). e_a must then be very small and its average $\langle e_a \rangle_\rho$ must be vanishing due to Ehrenfest theorem. On the other hand, for states sufficiently close to the stable thermo-dynamics equilibrium we are considering here, the right inequality of (3·29) guarantees that the constant Ω is obviously non-negative. Both yet unknown new quantities must be determined later.

Next, taking the divergence to both sides of Eq. (3·30), and evaluating along the particle's world line while imposing the left equation of (2·2), one gets

$$\Box U|_{\mathcal{C}(\lambda)} = mA|_{\mathcal{C}(\lambda)} + \gamma m U|_{\mathcal{C}(\lambda)}, \tag{3·31}$$

where, $A = (1/m)\partial^a e_a$ is some scalar, and $\gamma = 2\Omega/\Upsilon = 4\Omega/\hbar^2$ is some non-negative constant. Again, as for the case of the non-negativity of velocity divergence proven in the previous section, the condition (3·29) and Einstein local causality postulate that the particle's velocity must be less than the velocity of light, can be used to show that

$$\Box U|_{\mathcal{C}(\lambda)} \geq 0. \tag{3·32}$$

Let us note however that the above inequality is valid strictly only if the particle is sufficiently close to stable thermodynamics equilibrium states so that $\partial_a U|_{\mathcal{C}(\lambda)} \approx 0$. One thus must impose the right hand side of Eq. (3·31) to be also non-negative, locally, along the world history

$$\gamma U|_{\mathcal{C}(\lambda)} + A|_{\mathcal{C}(\lambda)} \geq 0. \tag{3·33}$$

The above inequality will be shown to play important key roles as we discuss the Hawking cosmological singularity theorem in the next next subsection.

Let us proceed to define a second rank tensors, A_{ab}, as $A|_{\mathcal{C}(\lambda)} = A_{ab} v'^a v'^b$, where v'^a is the tangent vector of the time-like geodesic, $\mathcal{C}(\lambda)$, of the curved space-time manifold. Putting this into Eqs. (3·31) and (3·27), one obtains the following relation

$$\gamma U_{ab} = R_{ab} - A_{ab}. \tag{3·34}$$

Hence, Eq. (3·34) tells us that the stochastic internal stress-energy-momentum tensor of the particle is proportional to the Ricci curvature of the corresponding effective curved space-time, corrected by some yet undetermined term. Next, recalling the local conservation of stochastic internal energy-momentum of Eq. (3·20), $\nabla'^a U_{ab} = 0$, and using the contracted Bianchi identity, $\nabla'^a R_{ab} - (1/2) g_{ab} \nabla'^a R = 0$, where $R = R^a_{\ a}$ is the curvature scalar, which is generally valid for any manifold with Lorentzian metric, g_{ab}, one obtains

$$\nabla'^a A_{ab} = \frac{1}{2} g_{ab} \nabla'^a R. \tag{3·35}$$

Integrating the above equation, one has

$$A_{ab} = (R/2 - \Lambda_E) g_{ab}, \tag{3·36}$$

where Λ_E is some constant. Finally, inserting Eq. (3·36) back into Eq. (3·34), one gets a "formally" similar equation with the celebrated Einstein field equation, where the conventional stress-energy-momentum tensor of the matter field, T_{ab}, is replaced by the "stochastic internal" stress-energy-momentum tensor of the particle, U_{ab}, as

$$\gamma U_{ab} = R_{ab} - \frac{1}{2} R g_{ab} + \Lambda_E g_{ab}. \tag{3·37}$$

Before proceeding to determine the value of the constant γ, let us discuss the physical nature of the so-called Einstein cosmological constant, Λ_E, which appears on the right hand side of Eq. (3·37). To do this, let us assume the effective curved space-time of the dynamics to be locally flat such that $g_{ab} = \eta_{ab}$, thus all Riemannian curvatures are locally vanishing, and in particular one has $R_{ab} = R = 0$. As discussed in the previous subsection, this situation corresponds to $U|_{\mathcal{C}(\lambda)} = U_{(i)}$, where $U_{(i)}$ is the stochastic internal energy corresponding to one of the stationary states which generates the locally flat effective curved space-time manifold. Eq. (3·37) then reduces into

$$\gamma [U_{(i)}]_{ab} = \Lambda_E^{(i)} \eta_{ab}, \tag{3·38}$$

where we have added an index "(i)" to the Einstein cosmological constant to denote that it corresponds to the stationary stochastic internal energy $U_{(i)}$. Since each $U_{(i)}$ is the bottom of a valley of the corresponding stable "local" thermo-dynamics equilibrium, then one can consider Einstein field equation with a discrete constant $\Lambda_E^{(i)}$ to describe the space-time dynamics of the sub-universe corresponding to the "local" stable thermo-dynamics equilibrium. In other words, each local stable equilibrium, or each sub-universe is identified with a specific Einstein cosmological constant. Furthermore, since for any time-like geodesic v'^a, $U_{(i)} = [U_{(i)}]_{ab} v'^a v'^b$ is positive definite, and $\eta_{ab} v'^a v'^b$ is negative definite, then one can conclude that $\Lambda_E^{(i)}$ must be negative definite. Hence, the existence of the Einstein cosmological constant with a negative definite value is a consequence of the demand on the definite positivity of stochastic internal energy, U. Since, as discussed in the previous section, the latter is equivalent to demanding the validity of Einstein local causality postulate, then the definite negativity of $\Lambda_E^{(i)}$ is guaranteed by and guarantees that $|v| < c$.

Now, let us discuss the magnitude of Einstein cosmological constant, $\Lambda_E^{(i)}$. To do this, recall that as proven in the previous section, in the Newtonian limit where

$|v| \ll c$, one has $U \approx mc^2/2$. On the other hand, since $|v| \ll c$, mainly only the time-time part of the stochastic internal stress-energy-momentum tensor contributes to the stochastic internal energy such that, $U \approx U_{00}(cdt/d\tau)^2 \approx U_{00}c^2$, where we have used particle's proper time as the affine parameter and the fact that in this regime one has $t \approx \tau$. Hence, comparing the above two facts, in Newtonian limit one has $U_{00} \approx m/2$. Inserting this into Eq. (3·38) one thus obtains

$$\gamma[U_{(i)}]_{00} = \gamma m_{(i)}/2 = \eta_{00}\Lambda_E^{(i)} = -\Lambda_E^{(i)}. \tag{3·39}$$

where, $m_{(i)}$ is the lowest possible mass that can be observed in the sub-universe "(i)". Hence, one can safely assume that the contribution of the cosmological constant in Eq. (3·37) is ignorable as compared to the other terms. Moreover, Eq. (3·39) indeed tells us that the cosmological constants correspond to the mass spectrum of elementary particles, as expected by Shakarov.[25),26)]

To determine the value of γ one can perform the Newtonian limit of our dynamics and compare it with Newtonian gravitational equation.[40)] First, in the limit of $v \ll c$, Eq. (3·16) gives us

$$-g_{00} = \frac{2}{mc^2}(U|_{\mathcal{C}(\lambda)} - U_{(i)}) + -\eta_{00}. \tag{3·40}$$

Using the relation between the mass and particle's stochastic internal energy, $U \approx mc^2/2$, valid in the limit $v \ll c$, the above relation can be put into a more intuitive form as

$$g_{00} = \eta_{00} + \frac{m_{(i)} - m}{m}. \tag{3·41}$$

It is thus clear that in the limit $v \ll c$, the difference between the particle's mass, m, and the lowest possible mass of the sub-universe, $m_{(i)}$, is the source of the curvature of the effective curved space-time corresponding to the local stable equilibrium "i".

Next, taking the spatial Laplacian, $\partial^2 = \sum_{i=1}^{3}\partial_i^2$, to both sides of Eq. (3·40), one has

$$-\partial^2 g_{00} = \frac{2}{mc^2}\partial^2 U|_{\mathcal{C}(\lambda)}. \tag{3·42}$$

Now, in the limit of $v \ll c$, the D'Alembertian operator becomes Laplacian operator such that from Eq. (3·31) one has $\partial^2 U|_{\mathcal{C}(\lambda)} \approx mA_{00}c^2 + \gamma m U_{00}c^2$. Inserting this into Eq. (3·42), one therefore gets

$$-\partial^2 g_{00} = 2(A_{00} + \gamma U_{00}). \tag{3·43}$$

Further, contracting Eq. (3·37) and ignoring the contribution of the Einstein cosmological constant, one has $R \approx -\gamma U_a^a \approx \gamma U_{00}$, where we have assumed that $|U_{aa}| \ll U_{00}$ for $a = 1, 2, 3$, which is valid for non-relativistic regime, $v \ll c$. Inserting this into Eq. (3·36), again ignoring the contribution of the Einstein cosmological constant and assuming that for the weak gravitational regime we are discussing, the Lorentzian metric is almost flat, $g_{ab} \approx \eta_{ab}$, one has $A_{00} \approx -(\gamma/2)U_{00}$. Putting this into Eq. (3·43), one thus gets

$$-\partial^2 g_{00} = \gamma U_{00}. \tag{3·44}$$

On the other hand, from the geodesic equation of (3·6), for $v \ll c$, one has[40] $g_{00} \approx -(1+2\Phi/c^2)$, where Φ is the Newtonian gravitational potential. Inserting this into Eq. (3·44), one therefore obtains

$$\partial^2\Phi = \gamma mc^2/4, \tag{3·45}$$

where we have put $U_{00} = m/2$. Finally, comparing the above equation with the Poisson equation for the Newtonian potential, $\partial^2\Phi = 4\pi G\rho_m$, where G is the Newton's constant and $\rho_m = m/c^2$ is the mass-energy density, γ in Eq. (3·37) has to be identified with the Newton's constant as

$$\gamma = 16\pi G/c^4, \tag{3·46}$$

that is twice larger than that of original Einstein field equation. However, since $T_{00} = m \approx 2U_{00}$ for weak gravitational field, one can expect that in this regime, our field equation of (3·37) will recover the predictions of Einstein field equation. Indeed, all the tested predictions of the Einstein field equation are so far performed in this weak field regime.[40]

Below, we shall proceed to assume that Eqs. (3·37) with (3·46) is the correct equation to describe the dynamics of space-time sufficiently near to states of stable thermo-dynamics equilibrium. Our assumption is not unfounded, due to the fact that in deriving the original Einstein equation, one must develop the local conservation of energy-momentum, $\nabla'^a T_{ab} = 0$, by invoking the results of another theory, say the quantum field theory, or otherwise assuming its validity from the beginning, ad-hocly.[9] Hence, in this sense, Einstein general relativity is not closed. By contrast, in our theory, the local conservation of "stochastic internal" energy-momentum, $\nabla'^a U_{ab} = 0$, is developed within the theory, as shown at the end of the previous subsection. Moreover, as discussed in the previous section, in the limit $|v| \ll c$ and neglectable mass fluctuations, U automatically gives us the Klein-Gordon equation. Thus, "some" quantum field theory might have already been included in our formalism of general relativity. Accordingly, later on, we shall call Eq. (3·37) with discrete spectrum of negative definite Λ_E as the Einstein field equation. As we proceed, we shall show that Eq. (3·37) is indeed more natural than the original Einstein field equation and in particular suffers no cosmological singularity as the original does.

At this stage, it is instructive to draw some conclusions concerning the Einstein field equation of (3·37). First, re-emphasizing, it is clear that Einstein field equation is a subset of flat space-time quantum-thermo-dynamics after being decoded in coordinate free geometrical terms. We thus have proven that gravity is indeed some thing not to be quantized. In particular, we have shown that Einstein field equation governs the dynamics of the "effective curved space-time" which is valid for the case when the state of the particle is very near to stable thermo-dynamics equilibrium. Moreover, each local stable thermo-dynamics equilibrium, or each sub-universe, is identified with different negative definite Einstein cosmological constant. In this sense, one can thus consider Eq. (3·25) as a generalized version of Einstein field equation, valid for any general states. In fact, the most important ingredient which reduces Eq. (3·25) into Einstein field equation of (3·37) is the requirement of Eqs.

(3·29) which restrict the particle to stay sufficiently close to a stable thermodynamics equilibrium state. These conditions in particular will constraint the value of Ω and thus of γ to be non-negative. In general, however, it is not always the case. For example, in state sufficiently near to "unstable" thermo-dynamics equilibrium, say in the vicinity of big bang, instead of Eqs. (3·29), one must impose

$$\partial_a U|_{\mathcal{C}(\lambda)} \approx 0, \quad \partial_a^2 U|_{\mathcal{C}(\lambda)} \leq 0. \tag{3·47}$$

Hence, one will again obtains Eq. (3·30) but now with a non-positive Ω and thus non-positive γ. We shall show later that this particular fact will keep the state of the big bang to be finite.

Moreover, our discussion on Newtonian limit suggests an ontology that as particle's mass m is large and thus its velocity is very small as compared with the velocity of light, $v \ll c$, then the stochastic internal energy, U, namely the particle's quantum-ness, will only appear in the time-time component of the corresponding effective curved space-time metric, g_{00}, such that the quantum-ness now is felt as gravitation through Eq. (3·45). In other words, Newtonian gravitation can be regarded as the remnant of the quantum-ness in the limit of large mass. In this way, the Newtonian "absolute time" emerges to acquire its privilege role to record the causal evolution of the dynamics.

Let us remark that, in one of the quantum-gravity programme called semiclassical quantum-gravity, one ad-hoc-ly assumes the Einstein field equation, whose matter source is now given by the quantum mechanical expectation value of the matter-field's stress-energy-momentum tensor. Hence, instead of Eq. (3·37), one postulates

$$\gamma_E \langle T_{ab} \rangle|_{\psi} = R_{ab} - \frac{1}{2} R g_{ab} + \Lambda_E g_{ab}, \quad \gamma_E = \gamma/2, \tag{3·48}$$

where, $\langle \ldots \rangle|_{\psi}$ is denoting the quantum mechanical expectation value over a quantum state ψ. Among all the programmes of quantum-gravity proposed so far, semiclassical quantum-gravity is the closest to ours. In fact all the theoretical prediction of black hole thermodynamics is firstly developed within the frame work of semiclassical quantum-gravity.[39] In the next subsection, as the first application of our reformulation of Einstein general relativity, we shall reproduce the celebrated Bekenstein-Hawking entropy, using simple physical assumption, proving that it is valid for any general effective curved space-time manifold even in the non-relativistic regime, and uncovers its physical meaning.

3.3. *Heisenberg Uncertainty Relation, Minimum Epistemological Length and the Ontology of Space-Time Entropy*

Let us apply our new theory to pounder some new light on long standing issues that are supposed to be relevant for any quantum theory of gravity. First, in this subsection, let us discuss the physical meaning of space-time entropy as represented by the celebrated Bekenstein-Hawking black hole entropy. To do this, let us show heuristically that the Heisenberg uncertainty principle if combined with the Einstein field equation will lead to the existence of a minimum uncertainty length. This of course is not new, and in fact is widely suspected as the general characteristic of any quantum theory of gravity.[41] One usually develops this theoretical statement by proving that if gravity is taken into account then instead of Heisenberg uncertainty relation, one will have the so-called generalized uncertainty relation[42),43)]

$$\Delta q^i \geq \frac{\hbar}{\Delta p^i} + L^2 \frac{\Delta p^i}{\hbar}, \tag{3·49}$$

where L is a constant of the order of Planck length, given by $L_P = (\hbar G/c^3)^{1/2}$. Interestingly, the functional form of the above relation is rather model independent. A direct consequence of the generalized uncertainty principle above is that the uncertainty in position has a minimum value given by $\Delta q^i_{\rm min} \approx 2L$.

However, since we have proven that general relativity is already included in the quantum-thermo-dynamics, the original Heisenberg uncertainty relation must be sufficient by itself. Below, using the original Heisenberg uncertainty relation, we shall show heuristically that it will lead to a minimum uncertainty length, in fact, having the same order as the one derived using the generalized uncertainty relation above. First, from the Heisenberg uncertainty relation, one has

$$h = 2\pi\hbar \leq \Delta p^i \Delta q^i = m\frac{\Delta v^i}{\Delta q^i}(\Delta q^i)^2 \sim m\theta^i(\Delta q^i)^2, \tag{3·50}$$

where for sufficiently small uncertainty in position Δq^i, $\theta^i \sim \Delta v^i/\Delta q^i$ is the $i-$part of the velocity divergence. The velocity divergence is thus bounded from below by the uncertainty in position as

$$\theta^i \geq \frac{2\pi\hbar}{m(\Delta q^i)^2}. \tag{3·51}$$

On the other hand, evaluating the continuity equation of (2·11) along the particle's world line, one has

$$\partial_\lambda \rho|_{\mathcal{C}(\lambda)} = -\partial_a(\rho v^a)|_{\mathcal{C}(\lambda)} = -\theta\rho|_{\mathcal{C}(\lambda)}, \tag{3·52}$$

where in the second equality we have used the left equation in (2·2). Integrating the above equation, θ^i then gives the rate of decay of ρ in the $i-$direction as

$$\rho \sim \exp(-\int^t \theta^i\, dt'). \tag{3·53}$$

Now let us consider a system of a particle with mass m. Einstein field equation tells us that an amount of mass m will generate an event horizon beyond which even light can not pass through. The radius of this event horizon is given by the Schwarzschild radius as[*)]

$$r_S = 2Gm/c^2. \tag{3·54}$$

*) In fact the Schwarzschild radius can also be obtained using Newtonian gravitational physics by defining the event horizon as the place where the kinetic energy of light, considering it as a particle with a finite mass, is equal to the gravitational potential exerted by the particle.

Let us make the following heuristic reasoning. Let us put an intuitive assumption that light is the most stable object in Nature such that the decay rate of the light's imaginability ρ_l is always faster than the decay rate of any particle's imaginability, ρ_p, such that one has

$$\rho_l \sim e^{-\int \theta_l\ dt'} \leq \rho_p \sim e^{-\int \theta^i\ dt'}, \tag{3·55}$$

where θ_l and θ are the velocity divergence of the light and the particle, respectively. In other words, light reaches its stationary states faster than anything else. What is then the velocity divergence of the light? Since light is trapped within the Schwarzschild radius then it is plausible to assume that the Schwarzschild radius, r_S, gives the uncertainty in position and the speed of light, c, gives the uncertainty in velocity, such that one has $\theta_l \sim \Delta v/\Delta q \sim c/r_S$. Inserting this into Eq. (3·55), one thus in general has

$$\theta^i \leq \theta_l \sim \frac{c}{r_S} = \frac{c^3}{2Gm}, \tag{3·56}$$

namely the particle's velocity divergence is bounded from above by the inverse of time needed by the light to travel along the distance of r_S. Combining Eqs. (3·51) and (3·56), one has $2\pi\hbar/m(\Delta q^i)^2 \leq \theta^i \leq c^3/2Gm$. One thus finally obtains

$$\Delta q^i \geq \sqrt{\frac{4\pi\hbar G}{c^3}} = 2\sqrt{\pi}L_P, \tag{3·57}$$

giving us the minimum uncertainty length of $\Delta q^i_{\mathrm{min}} = 2\sqrt{\pi}L_p$. Notice that the inertial mass of the particle m precisely cancels, leaving only constants of Nature!

Next, defining an infinitesimal area as $\Delta A = \prod_{i=1}^{2} \Delta q^i$, the minimum uncertainty in position will generate a minimum uncertainty in area as

$$\Delta A_{\mathrm{min}} \approx 4\pi L_p^2 = 4\pi A_P, \tag{3·58}$$

where $A_P = L_P^2$ is Planck area. Let us note that the minimum area appears as one makes observation. It tells us the limit of knowledge that we can obtain by performing measurements, thus is epistemological in nature. In other words, it is not an objective-ontological physical fact! In reality, space is continuum. In conclusion, one can never tell the physics inside the Planck scale. Or, any prediction concerning the physics inside a Planck scale is useless, in the sense that it will not be measurable reliably.

Further, we have proven in the previous section that the rate of change of the Boltzmann-Gibbs-Shannon entropy in non-relativistic regime can be written as[23)]

$$\frac{dS_e}{dt} = k_B \lim_{\sup\{\Delta V_{(j)}\}\to 0} \sum_j \rho(q^i_{(j)}) \frac{d(\Delta V_{(j)})}{dt},$$

where the summation is taken over all infinitesimal spatial dynamical volume or volume cell $\Delta V = \prod_{i=1}^{3} \Delta q^i$ of the particle. Here, each $q^i_{(j)}$ is a point inside the infinitesimal volume cell $\Delta V_{(j)}$. Next, since each volume cell tends to increase, for non surface cells, the tension to expand will be canceled by the tension coming from its neighborings. Thus, the expansion of the total volume can only come from the cells that compose the surface of the total dynamical volume. Now, for each surface/outer cell, let us define a constant $\mathcal{A}_{(j)}$ as

$$\rho(q^i_{(j)})\Delta V_{(j)} = \Delta A_{(j)}/\mathcal{A}_{(j)}, \tag{3·59}$$

where $\Delta A_{(j)}$ is part of the area of each outer cell which does not meet the surface of the other outer cells, composing the boundary of the total dynamical volume. The value of $\mathcal{A}_{(j)}$ must of course depend on the partition of the surface of the dynamical volume $\{\Delta V_{(j)}\}$, which is completely arbitrary. What does it mean? The left hand side certainly means the "number of times" that the particle would visit the outer volume cell $\Delta V_{(j)}$. The right hand side must thus reflects this physical fact. In other words, the right hand side must count "the number of micro-states" available in the dynamical area of $\Delta A_{(j)}$. This thus forces us to assume that $\mathcal{A}_{(j)}$ is independent of any partition and must be given by the area of a single microstate. The natural choice then is to "define" the area of a single microstate equal to the minimum uncertainty area given by the Heisenberg uncertainty relation and Einstein field equation of Eq. (3·58) as

$$\mathcal{A}_{(j)} = 4\pi A_P. \tag{3·60}$$

Inserting Eqs. (3·59) and (3·60) into (2·55), assuming that the state is sufficiently close to the stable thermodynamics equilibrium such that the time derivative of ρ can be ignored, one finally gets

$$\frac{dS_e}{dt} = \frac{k_B}{4\pi A_P}\frac{dA}{dt}, \tag{3·61}$$

where $A = \sum_{(j)} A_{(j)}$ is the total area of the surface of the dynamical volume of the particle. Let us pause a while, recalling that the second law guarantees the left hand side to be non-negative. This fact thus gives us

$$dA/dt \geq 0, \tag{3·62}$$

which states that the dynamical surface area of the particle is never decreasing, either constant of time for stable thermodynamics equilibrium states or monotonically increasing for near-equilibrium states. This then generalizes Hawking area law that the area of future event horizon of a black hole can never decrease,[17] to any general space-time structure.[23] Next, integrating both sides of Eq. (3·61), one obtains the celebrated Bekenstein-Hawking entropy

$$S_e = k_B\mathcal{N} + S_0, \quad \mathcal{N} = (1/4\pi)A/A_P. \tag{3·63}$$

where S_0 is some constant and $\mathcal{N}$ can be interpreted as the number of possible micro-states on the surface of the dynamical volume. Notice however that in contrast to the original Bekenstein-Hawking entropy which is derived for a black hole, our result of the linear relation between entropy and dynamical area is derived for general space-time. In fact, we have derived it in a non-relativistic regime. Hence, the linear proportionality between entropy and dynamical area must be considered as a fundamental relation expressing basic physical meaning of entropy. Now, it can be read directly from Eq. (3·63) that entropy counts the number of observable micro-states that are lying on the boundary/surface of the dynamical volume of the system, $\mathcal{N}$. Since this number is obtained using a scaling block of the minimum uncertainty area $\mathcal{A}_{(j)} = 4\pi A_P$ which is derived in the context of a quantum measurement, then this number has an information thus epistemological nature. In other words, BGS entropy tells the highest information content of a dynamical system that one can obtain through reliable measurement.

Next, still in the non-relativistic regime, inserting Eqs. (2·25) and (3·63) into the first law of equilibrium thermodynamics of Eq. (2·42), one has

$$\delta\bar{m} = \frac{1}{2}\frac{k_B T}{\pi A_P c^2}\delta A + \text{"work"}, \tag{3·64}$$

where $\bar{m} = \int d^3q\rho m$. Thus one has $\bar{m} \approx m$, $v \ll c$. Eq. (3·64) must remind us to the first law of black hole thermodynamics.[19] Yet, the first law of thermodynamics of (3·64) is now shown to be valid for any space-time structure. Note that we have derived these results using an approach which is not only much more clear physically, but also much more simple mathematically compared either to the way they were firstly derived using the semiclassical quantum-gravity, or to more complex and sophisticated tentative theories of quantum-gravity such as loop quantum gravity or string theory, which in turn lacking any clear physical interpretation. Moreover, in contrast to the predictions of these theories, our approach says that the Planckian physics takes place even in the non-relativistic regime, $|v| \ll c$.

3.4. *Rubbing-Off Cosmological Singularity: Non-singular Big Bang States*

Let us now apply our de-construction of general relativity to pounder some light onto one of the most disturbing foundational problems of Einstein general relativity, namely Einstein general relativity predicts that the universe might have begun from a singular initial state: The Hawking cosmological singularity. From the development of the Einstein field equation in the previous subsection, it is obvious that it satisfies the so-called energy conditions.[39] This last fact has been shown to play a prominent role in the development of singularity theorems in Einstein general relativity. First, the validity of the so-called "weak" energy condition is evidently clear from the basic postulate of the quantum-thermo-dynamics which assumes that the stochastic internal energy, U, is positive definite along the world line, $\mathcal{C}(\lambda)$, such that one always has

$$U|_{\mathcal{C}(\lambda)} = U_{ab}v'^a v'^b > 0. \tag{3·65}$$

Moreover, the so-called "strong" energy condition is nothing but the content of Eq. (3·33), which is valid in states sufficiently close to a stable thermodynamics equilibrium. In other words, using Eq. (3·33), Einstein field equation satisfies

$$R_{ab}v'^a v'^b = (\gamma U_{ab} + \frac{1}{2}Rg_{ab} - \Lambda_E g_{ab})v'^a v'^b = \gamma U|_{\mathcal{C}(\lambda)} + A|_{\mathcal{C}(\lambda)} \geq 0. \tag{3·66}$$

Still limiting to the state near to stable thermo-dynamics equilibrium, inserting Eq. (3·66) into Eqs. (3·27) and (3·21), one obtains the following important relation

$$\frac{d\theta}{d\lambda}\Big|_{\mathcal{C}(\lambda)} = -R_{ab}v'^a v'^b \leq 0, \tag{3·67}$$

which is well-known as focusing equation, whose prominent role in the singularity theorems will be clear soon. Hence, the divergence of the particle's velocity near to state of stable thermo-dynamics equilibrium is monotonically decreasing. This fact shows the attractiveness of gravity. Finally, let us note that in contrast to the Einstein general relativity where the energy conditions, both weak and strong, are assumed outside the theory, our de-construction of general relativity develops the energy conditions within the theory. In other words, it is these two conditions that guarantees the validity of Einstein field equation in the vicinity of stable thermodynamics equilibrium states. Hence, when we talk about Einstein general relativity, we (implicitly) assumed that energy conditions are satisfied. This fact will be shown to provide a quantum-thermo-dynamical loop-hole for the Hawking cosmological singularity theorem.

Further, before proceeding to discuss the cosmological singularity theorem, let us emphasize that, as clearly discussed in the derivation of Einstein field equation of (3·37), unlike the weak energy condition which is always valid in any thermo-dynamical states, the strong energy condition of Eq. (3·66) and thus the focusing equation of (3·67), are only valid strictly in the states "sufficiently near" to "stable" thermo-dynamics equilibrium. In general states, Eq. (3·67) does not hold. In particular, in the vicinity of "unstable" thermo-dynamics equilibrium states, as discussed in the previous section, the demand for $\partial_a U|_{\mathcal{C}(\lambda)} \approx 0$, $\partial_a^2 U|_{\mathcal{C}(\lambda)} \leq 0$ and the Einstein local causality postulate, $|v| < c$, will lead to the fact that $\Box U|_{\mathcal{C}(\lambda)} \leq 0$, such that from Eq. (3·27), one will obtain

$$R_{ab}v'^a v'^b \leq 0. \tag{3·68}$$

Hence, in this case, instead of the focusing equation of (3·67), one has

$$d\theta/d\lambda \geq 0, \tag{3·69}$$

say the accelerated diverging equation. In these states, gravitation must be felt as repulsion. Moreover, in this regime, the dynamical volume of the particle is expanding in an accelerated exponential rate.[23)]

Penrose and Hawking then used the focusing equation of (3·67) to prove their theorems for the existence of singularity in space-time, at which, at least a geodesic cease to exist. The idea of the singularity theorems utilizes the following fact: suppose at one instant of affine parameter, say λ_0, the velocity divergence is negative, $\theta(\lambda_0) < 0$, then using Eq. (3·67), the divergence must be minus infinity in a finite interval of affine parameter, $\theta(\lambda) = -\infty$, where $|\lambda - \lambda_0|$ is finite. Of course, this only signifies that a caustic will develop at a space-time point, if $\theta < 0$ any where in space-time manifold. However, Penrose and Hawking then proceeded to show that this fact if combined with the global properties of some space-time manifold will lead to the existence of space-time singularity in a finite time in the past, *i.e*, in the beginning of the universe which is proven by Hawking,[8] and in the future directed gravitational collapse, which historically preceded the Hawking cosmological singularity theorem, and proven by Penrose.[7] In particular, the possibility of space-time singularity in the beginning of our universe, is unacceptable since then it is causally connected to our present and future history, and thus naked. Below we shall thus only discuss this type of space-time singularity.

Now, let us spell-out the conditions for the proof of the existence of cosmological singularity in the beginning of our universe, and show, using our reformulation of general relativity that one of those conditions is no more valid. The theory can be stated as follows:[39]
"Let us assume that our universe is a globally hyperbolic space-time with (i) the strong energy condition and Einstein field equation are valid such that $R_{ab}v'^a v'^b \geq 0$ for all time-like v'^a, hence the focusing equation is valid, $d\theta/d\lambda \leq 0$, and (ii) at an instant of λ, the universe is expanding everywhere. Then the universe must have begun in a singular state a finite time ago".
Indeed, as shown in the previous section, due to the second law of thermodynamics expressed in Eq. (2·45), the universe must be non-contracting, that is for future directed time-like v'^a, one must have $\theta \geq 0$. Observation indeed confirmed that the universe is in fact expanding, thus $\theta > 0$.[44] Hence, the condition (ii) is satisfied. This means that for the past directed time-like v'^a, one has $\theta < 0$. Thus, had the condition (i) been true, then at a finite past, a caustic must develop, which signifies the existence of space-time singularity in a finite past.

However, although the quantum-thermo-dynamics and also the observation support the validity of the condition (ii), the validity of the condition (i) for past directed time-like geodesic is questionable. In fact, as the universe goes to the past, the second law of thermodynamics must guide the universe to move away from the stable thermo-dynamics equilibrium state, such that, as discussed in the previous paragraph, the strong energy condition and thus Einstein field equation are no more valid. Hence, as the universe is sufficiently far from any stable thermodynamics equilibrium state, in general one has $R_{ab}v'^a v'^b \ngeq 0$, and thus $d\theta/d\lambda \nleq 0$. In particular, at the beginning of the universe, say in the vicinity of the big bang, the state of the universe must be very close to an "unstable" thermo-dynamics equilibrium state such that, as discussed in the previous paragraph, instead of the focusing equation, one has the accelerated diverging equation, $d\theta/d\lambda \geq 0$. Hence, before θ becomes minus infinity, there must be a turning point λ_t at which $d\theta/d\lambda = 0$ and changes sign afterward, $d\theta/d\lambda > 0$. This in turn will drag the past directed velocity divergence upward till it is vanishing, $\theta = 0$, thus the universe stop contracting. We have thus an early universe which is extremely dense but still finite. In this situation, gravity is felt as repulsion. In this way, Hawking cosmological singularity might be smeared out: there is a big bang with no singularity.

Conversely, rolling the story forward in time. The universe must have begun from an extremely dense and unstable big bang state. "Something" then might happen that perturbed this unstable equilibrium state such that it started to expand, $\theta > 0$. Since in this regime $d\theta/d\lambda \geq 0$, then the expansion rate must be accelerating. Then after a period of interchanging between $d\theta/d\lambda \leq 0$ and $d\theta/d\lambda \geq 0$, with θ is kept positive thus the universe keep expanding, it eventually will reach an area sufficiently close to one of a local stable equilibrium state, where $d\theta/d\lambda \leq 0$. This will drag the expansion rate to be eventually vanishing, $\theta = 0$, at which the universe will reach the stable thermodynamics equilibrium or quantum mechanical stationary state. It will be shown later that once a (closed) universe reaches an equilibrium state, it will be trapped there forever. Since in thermodynamics equilibrium states entropy is constant, $dS_e/d\lambda = 0$, then "biological time" must stop to exist, in the sense that time does no more create new information.

3.5. *Space-time quantization and Poincare Group*

One lesson that we learn from the above discussion is that, once the particle enters in the area of the vicinity of a stable thermo-dynamics local equilibrium state, or a sub-universe, then it is dragged by the gravity through the focusing equation, $d\theta/d\lambda \leq 0$, till the particle velocity is divergence-less

$$\theta = \partial_a v^a = \nabla'_a v'^a = 0. \tag{3·70}$$

Using this fact, one has

$$\frac{d\rho}{d\lambda}\Big|_{\mathcal{C}(\lambda)} = \partial_\lambda \rho|_{\mathcal{C}(\lambda)} + v^a \partial_a \rho = -\rho\, \partial_a v^a = 0, \tag{3·71}$$

where in the second equality we have insert the continuity equation of (2·11). Thus the particle's imaginability along the world line is constant of motion. This eventually leads to the fact that the particle's stochastic internal energy is locally also constant of motion

$$\frac{dU}{d\lambda}\Big|_{\mathcal{C}(\lambda)} = 0. \tag{3·72}$$

This, as discussed in the previous section, will lead to quantization $U|_{\mathcal{C}(\lambda)} = U_{(i)}$, $i = 1, 2, 3\dots$, where $U_{(i)}$ is the eigenvalues of $\spadesuit = -\Box$. Moreover as discussed in the previous section, in this case the Boltzmann-Gibbs-Shannon entropy is constant, $dS_e/d\lambda = k_B\langle\theta\rangle_\rho = 0$, thus the particle has reached one of its stable thermodynamics equilibrium states, and ρ takes the form of Maxwell-Boltzmann-Gibbs canonical distribution.

Hence, in the state of stable thermo-dynamics equilibrium, the matter source on the left hand side of the Einstein field equation of (3·37) can only take discrete values and constant along the particle's world line, $\mathcal{C}(\lambda)$. This thus suggests that the Lorentzian metric of the effective curved space-time in the stable thermo-dynamics equilibrium states must also have a discrete nature and also be constant of motion, locally. To see the last point more clearly, inserting the stationary stochastic internal energy $U|_{\mathcal{C}(\lambda)} = U_{(i)}$ into Eq. (3·9), one has

$$-g^{(i)}_{ab} v'^a v'^b = \frac{2}{m} U_{(i)} + D|_{\mathcal{C}(\lambda)}. \tag{3·73}$$

Here, $g^{(i)}_{ab}$ is the Lorentzian metric of the effective curved space-time corresponding to the stationary stochastic internal energy $U_{(i)}$. Taking the Lie derivative along the geodesic, $\mathcal{L}_{v'}$, to both sides, noticing that $U_{(i)}$ and $D|_{\mathcal{C}(\lambda)}$ are constant along the geodesic and the fact that $\mathcal{L}_{v'}v'^a = [v', v']^a = 0$, one finally obtains

$$\mathcal{L}_{v'}g^{(i)}_{ab} = 0. \tag{3·74}$$

The above equation is nothing but a killing equation, such that the geodesic v'^a is a killing vector.

On the other hand, taking the Lie derivative of a Lorentzian metric one has[45)]

$$\mathcal{L}_{v'}g_{ab} = v'^c\partial'_c g_{ab} + g_{ac}\partial'_b v'^c + g_{cb}\partial'_a v'^c = \nabla'_a v_b + \nabla'_b v_a. \tag{3·75}$$

Taking the trace of both sides, one thus obtains

$$g^{ab}\mathcal{L}_{v'}g_{ab} = 2\nabla'^a v'_a = 2\theta. \tag{3·76}$$

Hence, if there is a killing vector v'^a whose integral curve makes the particle's geodesic, then using metric representation of the velocity divergence above, one has

$$\theta = \nabla'_a v'^a = \frac{1}{2}g^{ab}\mathcal{L}_{v'}g_{ab} = 0, \tag{3·77}$$

which shows that the particle has reached one of its local stable thermo-dynamics equilibrium states. In this case, the metric must then take the form of one of the stationary metric $\{g^{(i)}_{ab}\}$ corresponding to the stationary states $\{U_{(i)}\}$. Eq. (3·74) and (3·77) thus show us that the tangent vector $d\mathcal{C}/d\lambda$ of the particle's world line in the stable thermo-dynamics equilibrium states will generate a one parameter isometry ϕ, $\phi^* g^{(i)}_{ab} = g^{(i)}_{ab}$[39)] and conversely, any symmetries observed in Nature thus reflect that it has reached a thermodynamical state. In other words, the particle will end up possessing local symmetries of its effective curved space-time while reaching its stable thermo-dynamics equilibrium state. This is the dynamical origin of the ubiquitous symmetries that we observe in the universe.

Further, since g_{ab} of the effective curved space-time corresponds to the stochastic internal energy, U, in flat space-time, then any symmetry in curved space-time must reflect the symmetry in its corresponding stochastic internal energy. Hence, in stable thermo-dynamics equilibrium states, the stochastic internal energy will locally be symmetric. To study what sorts of local symmetry the effective curved space-time or the stochastic internal energy of the particle fall into, let us note that in a stable thermodynamics equilibrium state, one has $\partial^a U_{(i)} = 0$. Inserting this into the dynamical equation in the original flat space-time of (2·10), one has

$$\frac{dv^a}{d\lambda} = 0. \tag{3·78}$$

As is shown in the previous section, in the effective curved space-time coordinate system this leads to

$$\frac{dv'^a}{d\lambda} = 0, \tag{3·79}$$

hence, the affine connection of the corresponding effective curved space time is locally vanishing, $\Gamma^a_{bc} = 0$. In other words, the corresponding effective curved space-time is locally flat, $g_{ab}^{(i)} = \eta_{ab}$. One can of course arrive directly at all these conclusions by putting $U|_{\mathcal{C}(\lambda)} = U_{(i)}$ into Eq. (3·16). Since the killing vectors of the Minkowskian metric, η_{ab}, are the generators of the Poincare groups,[40] hence, in stable thermodynamics equilibrium states, the particle will move in an effective curved space-time which locally possesses at least one of the symmetry belonging to the Poincare group. In this way, one recovers the special relativity as the local stable thermo-dynamical equilibrium limit of the general relativity. Moreover, since quantum field theory is developed as the representation of Poincare group, then one can expect that quantum field theory is basically valid only at stable thermodynamics equilibrium states. This is just another reincarnation of the fact we derived in the previous section, namely in the stationary states or in stable thermodynamics equilibrium states, one has the Klein-Gordon equation for $\mathcal{I}_{(i)}$.

3.6. *Penrose's Objective Wave Function Collapse*

First, as discussed in the previous section, since the stationary imaginability $\{\mathcal{I}_{(i)}\}$ makes a complete set of orthonormal functions,[37] then before the stable thermo-dynamics equilibrium is reached, the particle's non-equilibrium or non-stationary imaginability can be expanded as

$$\mathcal{I}_{\rm neq}(q;\lambda) = \sum_i r_{(i)}(\lambda)\mathcal{I}_{(i)}(q),$$

where $r_{(i)}$ is some complex function. This is the superposition principle of the orthodox quantum mechanics. Notice that since each $\mathcal{I}_{(i)} = \sqrt{\rho_{(i)}}$ generates an effective curved space-time manifold with the Lorentzian metric $g_{ab}^{(i)}$, then the above superposition is formally a superposition of many universes, or many sub-universes. Hence we have come to a "formal" Everett-DeWitt's many universes[46),47] interpretation of quantum mechanics for dealing with quantum cosmology. However, as we discussed in detail in Ref.,[22] the superposition of Eq. (3·80) can not be considered as objective-ontological and has no physical nature. It is only a mathematical construct and thus is formal-epistemological. In reality, at any instant, our particle will only live in one universe with a determinate/definite metric g_{ab} generated by the particle's imaginability $\mathcal{I}$ with a degree of occurrence $\rho|_{\mathcal{C}(\lambda)} = |\mathcal{I}|^2|_{\mathcal{C}(\lambda)}$, which is determined by the initial condition of our single particle system. Moreover, since at any instant g_{ab} is definite, then one has a "definite space-time dynamical causal structure". Hence, in this sense one does not face the problem of "probabilistic causal structure", that is one important aspect of the problem of time, which is unavoidable if one wants to quantize the space-time geometry in a formal way, along with the spirit of the orthodox quantum mechanics.[11,12]

Finally, in the state of stable thermo-dynamics equilibrium, the metric of the effective curved space-time manifold of the system will land on one of the locally flat Lorentzian metric $g_{ab}^{(i)} = \eta_{ab}$, corresponding to one of the stationary imaginability $\mathcal{I}_{(i)}$. Since the dynamics at sufficiently near to stable thermo-dynamics equilibrium state is governed by the Einstein field equation, then the formal collapse of $\mathcal{I}_{\rm neq}$ onto $\mathcal{I}_{(i)}$ or g_{ab} onto $g_{ab}^{(i)}$ can be regarded as induced by gravitation. This, though not the precise idea, is the spirit of the Penrose's objective reduction of wave function (Penrose's OR).[13] Moreover, in the stable thermo-dynamics equilibrium state, since the effective curved space-time is locally flat, then the velocity divergence is constant of motion,

$$d\theta/d\lambda = -R_{ab}v'^a v'^b = 0. \tag{3·80}$$

Since in these states the velocity divergence is also vanishing, $\theta = 0$, then one can conclude that once the particle reaches the state of stable thermo-dynamics equilibrium, hence $\theta(\lambda_s) = 0$ for some affine parameter λ_s, then it will be trapped there forever, namely $\theta(\lambda) = 0$, for $\lambda \geq \lambda_s$.

§4. Summary and Conclusions

First, we have followed the insight that the key to a consistent quantum theory of gravity, quantum-gravity, might first begin with curing the foundational problem of the orthodox formalism of quantum mechanics:[14),48)] in particular the "epistemological nature" of the orthodox quantum mechanics and its variant of de-coherence programme,[49),50)] which reveals in the need of external observer/environment to collapse the wave function and brings to us a specific (classical) physical reality from a vast quantum potentialities. Assuming a flat Minkowskian space-time background, we have thus reformulated an objective-ontological quantum theory which has a spontaneous and internal mechanism for wave function collapse.[22),23)] It turned out that the new theory of quantum mechanics can be used to prove all the four laws of thermodynamics even for a single particle system.[23)] In fact, the origin of the spontaneous wave function collapse is provided by the second law of thermodynamic: any non-stationary wave function will be dragged by the law of increasing entropy to eventually land on one of the possible stationary states, corresponding to state of thermodynamics equilibrium. In this state, the entropy is maximized and classicality then rules. Thus the so-called quantum and thermal fluctuations are essentially two names for one single physical entity. This convinces us that the new unified theory of quantum mechanics and thermodynamics, quantum-thermo-dynamics, is appropriate for the quantization of gravity and especially for describing quantum cosmology.

However, rather than quantizing gravity directly by exercising the quantum-thermo-dynamics of a curved space-time, we have proven in the present paper that Einstein general relativity can be obtained from the flat space-time quantum-thermo-dynamics by restricting the latter in the vicinity of stable thermodynamics equilibrium states and extracting its geometrical content. In short, gravity has already been included in flat space-time quantum-thermo-dynamics. This is done by noticing that "the principle of maximal imaginability" which is used to develop the flat space-time quantum-thermo-dynamics, is in fact equivalent to "the principle of equivalence" which gives the very foundation of Einstein general relativity. The stochastic internal energy potential U of the flat space-time quantum-thermo-dynamic is shown to generate a Lorentzian metric g_{ab} of a curved space-time, on which the particle under consideration can be seen as a freely falling body. Hence, the Lorentzian curved space-time is not physically fundamental but is "phenomenological" or "effective".

Moreover, we have also developed a general relation between the stochastic internal energy of the flat space-time quantum-thermo-dynamics, U, and the Ricci curvature tensor, R_{ab}, of the corresponding effective curved space-time such that restricting to the vicinity of stable thermodynamics equilibrium states will lead to the celebrated Einstein field equation with negative definite Einstein cosmological constants. The definite negativity of the cosmological constant is shown as the consequence of Einstein local causality postulate. Unlike the original Einstein field equation, in the new formalism, the source of the gravity is not the stress-energy-momentum tensor, T_{ab}, but the "stochastic internal" stress-energy-momentum tensor, U_{ab}, where $U = U_{ab}v'^a v'^b$, and v'^a is the particle geodesic along the effective curved space-time. In non-relativistic regime, both are related as $T_{00} = 2U_{00} = m$. U_{ab} then couples to the Einstein tensor with coupling constant twice larger than that of the original Einstein field equation. In addition, each local equilibrium is characterized by different and discrete values of Einstein cosmological constants, corresponding to the spectrum of masses of elementary particles. In this sense, gravity is thus shown to be an "emergent phenomena" or the "remnant of quantum-ness" in the vicinity of stable thermodynamics equilibrium states. Our approach thus suggests that the programme to "quantize gravity" might be misleading.

We then apply the new formalism of general relativity, first to the thermodynamics of space-time to show that, even in the non-relativistic limit, $|v| \ll c$, the Boltzmann-Gibbs-Shannon entropy is linearly proportional to the dynamical surface area of the system. The relation is shown to be valid for general structure of effective curved space-time manifold. Our result thus generalizes the Hawking area law for the event horizon of a black hole and Bekenstein-Hawking entropy formula for black hole. This is done by noticing the fact that the Heisenberg uncertainty relation if combined with Einstein field equation will lead to the existence of a minimum uncertainty length which eventually gives us a minimum area below which one can not perform a reliable measurement. Using this minimum uncertainty area to scale the surface of the dynamical volume of the system, the entropy can now be interpreted as the number of measurable micro-states lying on the surface of the dynamical volume that the particle visits during the evolution. Hence, it gives the highest information content on the dynamical system that one can observe in reliable measurement. We have also derived the first law of space-time thermodynamics, which is in similar functional form as the first law of black hole thermodynamics.

Next, we have shown that the new formalism of general relativity can provide a mechanism to rub-off the Hawking cosmological singularity at the beginning of universe. This is seen directly from the fact that the strong energy condition and thus Einstein field equation and the focusing equation necessary for the occurrence of the cosmological singularity is valid only in states sufficiently close to stable thermodynamics equilibrium. Hence, since state in the vicinity of big bang must be very unstable, instead of the focusing equation, one must have the accelerated diverging equation of (3·69). This will keep the value of the past directed velocity divergence θ finite and eventually make it vanishing at the beginning of the universe. The universe must have begun from an extremely dense yet non-singular big bang state.

We also claims that our new approach of quantum-gravity does not face "the problem of probabilistic causality structure", or the problem of stochastic light cone, that is one of the most important aspect of the so-called problem of time. This is obvious since the new theory of quantum-thermo-dynamics shows that the principle of superposition of orthodox quantum mechanics is "formal" referring to no physical reality. At any instant, the particle has a definite ρ thus a definite U which then generates a definite Lorentzian metric g_{ab}, hence giving us "a definite causality structure". We also showed that the definite positivity of U will guarantee Einstein local causality postulate, such that one will always have a fixed direction of causality order. Finally, we have also shown that this fact is equivalent to the second law of thermodynamics.

Next, we have shown that while the dynamics reaches one of its stable thermodynamics equilibrium state, the effective Lorentzian metric collapses, due to gravity, onto one which is locally flat. Moreover, in stable thermodynamics equilibrium state, the world line of the dynamics will become the killing vector of the locally flat Lorentzian metric, thus comprising the generator of the Poincare group. In this way, one recovers the symmetry of the special relativity. Our approach thus clarifies that first, the wave function collapse is indeed induced by gravity. Second, the special relativity is shown as the stable thermodynamics equilibrium limit of general relativity. Moreover, in general non-equilibrium states, we have shown that the conventional relation between energy, momentum, and mass is no more valid, and must be corrected by non-trivial terms, which is no more ignorable for high energy elementary particles.

Finally, let us comment on experimental facts. It is usually assumed that a quantum theory of gravity is relevant only at Planck scale. This involves a length scale of the order of Planck length $\sqrt{\hbar G/c^3} \approx 1.62 \times 10^{-35}$ m and mass scale of the order of Planck mass $\sqrt{\hbar c/G} \approx 1.22 \times 10^{19}$ GeV. We have thus a situation in which matter is extremely dense such that gravity is microscopically no more ignorable. Two examples of this situation is believed to be relevant: early universe and black hole. Viewing this way, it is almost hopeless to at least obtaining facts with direct bearing on quantum-gravity. This also gives the main reason why the issue of constructing a quantum theory of gravity is still so much open. Our approach however showed that the above apparently plausible reasoning is simply misleading, and could trap oneself into a circle of despair: when one calls something as physical facts, one is implicitly employing a physical theory to organize data. Hence, there is no a clear cut line which separates "the supposed facts" from "a theory". In other words, when one tries to construct a physical theory, one is basically searching for a new way of organizing/selecting data into physical facts. Indeed, our approach has led us to "a new fact" that what we traditionally called as gravity is the remnant of quantum in the limit of large mass, so that the belief that quantum and gravity will only take place in the Planckian scale is simply misleading. We expect that our new approach thus will lead to new methods of experiments, involving ordinary scale of space-time and matter, to witnessing the intermingling effects of what we traditionally called as quantum mechanics, thermodynamics and gravitation. We leave it for future work.

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